CONTROL OF A NONLINEAR SYSTEM BY FUZZY SUPERVISION
A LOCAL APPROACH

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ABSTRACT: In this work, we propose a general method of a nonlinear control system by fuzzy supervision of a conventional control. This approach belongs to the class of methods based on the use of the linearized system. The particularity of this work lies in the proposed algorithms to realize the parametric adaptation of control law and running equilibrium estimation. These two aspects have been solved by the use of a fuzzy supervisor based on a Takagi-Sugeno rules. The rules bases are built by the knowledge of the gains and the equilibrium, for a fixed number of operating points. The gains are calculated using a local synthesis by poles placement. The implementation of this method is presented on a real example for which a generator of reference trajectory is presented. The results available are compared with the ones obtained by use of Isidori linearizing control method and lead to the validation of this method.

KEYWORDS: Fuzzy supervisor, fuzzy inference system, fuzzy parametric adaptation, fuzzy estimation of the equilibrium point, non linear system, first order control, pole placement, reference trajectory.

1. INTRODUCTION

The modeling of a real system practically always leads to a nonlinear model. When the operating range of the system is restricted, the use of a linearized model around an equilibrium point is sufficient to carry out the synthesis of a control law (D'Andrea-Novel, 1994). This approach is not possible when the system evolves on a large operating range. In the following it is assumed that the knowledge of one linearized model around an equilibrium point is not sufficient to satisfy the control objectives. To overcome this difficulty, several linearized models around various operating points are used.

The problem is then the adaptation of the control law according to the current equilibrium point. In the next we proposes a simple technique of the parameters adaptation of the control law based on a combined approach using the conventional control techniques and fuzzy control. This combined approach allows the development of a two-levels structure: a system level in which acts the control law and a supervision level allowing to adapt the parameters of the control according to the operating conditions. With the same objective, the Tanaka approach consists in building a fuzzy model of the system and a fuzzy control law by state feedback. This approach carries out an interpolation of the control laws but not of the parameters.

However, this control law is locally linear and function of current equilibrium. To ensure the stability of the closed-loop system, Tanaka (Tanaka and al. 1998) proposes to use a Lyapunov approach leading to the resolution of linear matrix inequalities (LMI). Compared to the approach suggested here, the Tanaka method also requires an estimate of the current equilibrium point. However the calculations are more complex and do not allow to determine the matrix of gain in order to impose the predefined performances on the closed-loop system. See also the works of Kuipers and Aström (1994) allowing commutation between various control laws according to the operating range in which the system evolves. A control law is associated at each operating range and characterized by a membership function. The control applied to the system results from a weighted average of the various control laws. But the authors underline the very delicate theoretical analysis of this structure. This approach to fuzzy control was pioneered by Takagi and Sugeno (1985) and Sugeno and Kang (1986).

The work presented here is divided into four sections. The first section introduces the concept of supervised system. The role of the various components are outlined. In the second section, the general structure of a fuzzy supervisor control is presented. The core of this unit: the fuzzy inference system (FIS) is briefly reviewed. In the third section a general method of fuzzy supervised control of a nonlinear system on a large operating range is proposed. In the fourth section an application concerning the control by the inductor of a D.C. current machine fed in voltage is presented. For this system, a generator reference trajectory is briefly presented. The performances of the supervised control will be then compared with the results obtained using a linearizing control of Isidori. The results available are compared with the ones obtained by use of Isidori linearized method and lead to the validation of this method.

2. GENERAL STRUCTURE OF A SUPERVISED SYSTEM

The general structure of a supervised system is presented on figure 1. This structure shows two levels. A level system, or level 0, whose role is primarily to generate the control law applied to the process in order to satisfy the control objectives. A higher level, called level 1, whose role is the decision-making from the measures issued from level 0. More precisely, the task of level 1 is ensured by a supervisor, consisting of an observation function Γ of the level 0 and a decision-making function Ψ, allowing to act on level 0. The role of the observation function Γ is to generate, from the variable of level 0, information on the behavior of the closed-loop system. The nature of this information is related to the goals to reach.

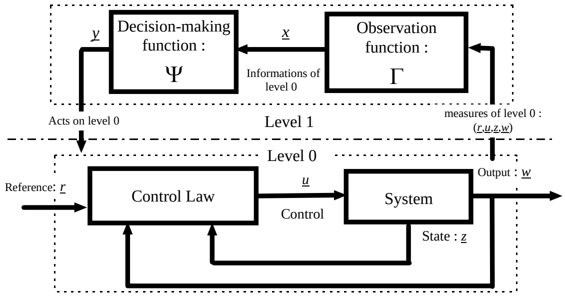

Fig. 1. General structure of a supervised system.

For example, if the aim is the conformity of the closed-loop system with a model of behavior, the function of observation generates a variation of this behavior. If if the aim is the diagnosis of failures, the observation function generates indicators of defects on the elements of the system. Usually, observation function generate all informations useful for the decision-making function. The data generated by the observation function are then processed by the decision-making function which generates the correct actions necessary for the satisfaction of the objectives. In the next, the correct actions undertaken are the parametric adaptation of the control law in order to satisfy the control objectives on a large operating range. Figure 2 gives the general constitution of the fuzzy supervisor used in the following. The function of decision-making is fulfilled here by a fuzzy inference system (FIS).

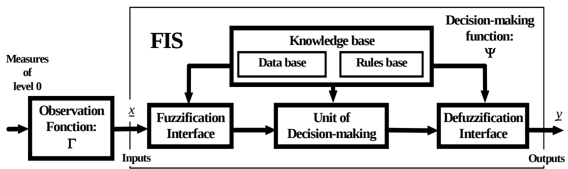

Fig. 2. Structure of a decision-making function.

Generally, the FIS is composed of four main blocks: a knowledge base, an interface of fuzzification, a unit of decision-making and an interface of defuzzification. For a more detailed study consult in particular (Foulloy, 1994; Driankov, 1996; Lee, 1990). Many examples of applications are found in the collective works (Sugeno, 1992; Kandel, 1994). The rules base of the FIS is a list of fuzzy conditional clauses. In the work which follows, only rules of the Takagi-Sugeno type are used. The rules base is written below:

$$\begin{cases} R_p\text{: If } \left(\underline{x} \text{ est } \underline{\tilde{\mathbf{A}}}_p\right) \text{ Then } \underline{y} = \underline{b}_p \text{ with } p = 1\ldots l \\ \underline{x} = \begin{bmatrix} x_1 & \cdots & x_{\dim(\underline{x})} \end{bmatrix}^T ; \underline{\tilde{\mathbf{A}}}_p = \begin{bmatrix} \tilde{\boldsymbol{a}}_1^p & \cdots & \tilde{\boldsymbol{a}}_{\dim(\underline{x})}^p \end{bmatrix}^T \\ \underline{y} = \begin{bmatrix} y_1 & \cdots & y_{\dim(\underline{y})} \end{bmatrix}^T ; \underline{b}_p = \begin{bmatrix} b_1^p & \cdots & b_{\dim(\underline{y})}^p \end{bmatrix}^T \end{cases} \quad (1)$$

Where l is the total number of rules, $\tilde{a}_i^p$ are the fuzzy subset associated to the input variable x_i.

The fuzzy subset $\tilde{a}_i^p$ are defined using memberships functions noted $\mu_{\tilde{a}_i^p}(x_i)$. The data base contains all the parameters of the membership functions. In the next, the fuzzy subset $\tilde{a}_i^p$ is defined using triangular membership functions. The output vector $\underline{y}$ is then calculated by the following relation:

$$\underline{y}(\underline{x}) = \frac{\sum_{p=1}^{l} \prod_{i=1}^{\dim(x)} \mu_{\tilde{a}_i^p}(x_i)\underline{b}_p}{\sum_{p=1}^{l} \prod_{i=1}^{\dim(x)} \mu_{\tilde{a}_i^p}(x_i)} \qquad (2)$$

3. PRINCIPLE OF THE METHODE

Figure 3 gives the general schem of a nonlinear control system, where $\underline{u}$ is the control vector, $\underline{z}$ is the state vector of the system, $\underline{v}$ is the state vector of the integrators intended to cancel the steady state error between the reference vector $\underline{r}$ and output $\underline{w}$.

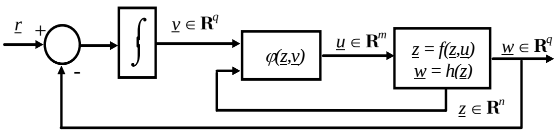

Fig. 3. Structure for the nonlinear control system.

The reference vector is assumed constant or slowly variable. φ is an unknown function used to generate the control from the state system and the integral of the error. The closed-loop system is then given by the following equations :

$$\begin{cases} \dot{\underline{z}} = f(\underline{z},\underline{u}); \quad \underline{u} = \varphi(\underline{z},\underline{v}) \\ \dot{\underline{v}} = \underline{r} - h(\underline{z}); \quad \underline{w} = h(\underline{z}) \end{cases} \qquad (3)$$

The problem of the synthesis of φ in a nonlinear context is difficult. However, it is possible to search φ locally by considering its linearized on the equilibrium space. The control law is then given by :

$$\underline{u} = \frac{\partial \varphi}{\partial \underline{z}}(\underline{z}_0,\underline{u}_0).(\underline{z}-\underline{z}_0) + \frac{\partial \varphi}{\partial \underline{v}}(\underline{z}_0,\underline{u}_0).(\underline{v}-\underline{v}_0) + \underline{u}_0 \qquad (4)$$

where $\underline{z}_0$, $\underline{v}_0$, $\underline{u}_0$ caracterize the equilibrium of the closed-loop system and are solutions of :

$$\begin{cases} f(\underline{z},\underline{u}) = 0 \\ \underline{r} - h(\underline{z}) = 0 \end{cases} \qquad (5)$$

Note that $\underline{v}_0$ can be taken equal to zero because it do not appear in the equilibrium conditions. The control law thus built is used to make evolve the system along the equilibrium tajectories defined by the equations (5). The system can be then represented by its linearized model. The parameters of the linearized model are functions of the current equilibrium point.

The linearized model of the closed-loop system is written:

$$\begin{cases} \dot{\underline{z}}^* = \frac{\partial f}{\partial \underline{z}}(\underline{z}_0,\underline{u}_0)\cdot\underline{z}^* + \frac{\partial f}{\partial \underline{u}}(\underline{z}_0,\underline{u}_0)\cdot\underline{u}^* \\ \dot{\underline{v}} = -\frac{\partial h}{\partial \underline{z}}(\underline{z}_0)\cdot\underline{z}^* = \underline{w}_0 - \underline{w} = -\underline{w}^* \\ \underline{u}^* = \frac{\partial \varphi}{\partial \underline{z}}(\underline{z}_0,\underline{u}_0)\cdot\underline{z}^* + \frac{\partial \varphi}{\partial \underline{v}}(\underline{z}_0,\underline{u}_0)\cdot\underline{v} \end{cases} \qquad (6)$$

with $\underline{z}^* = \underline{z} - \underline{z}_0$ and $\underline{u}^* = \underline{u} - \underline{u}_0$, $\underline{w}_0$ is the desired output at the equilibrium, that is $\underline{w}_0 = \underline{r}$. Let use consider:

$$\begin{cases} \frac{\partial f}{\partial \underline{z}}(\underline{z}_0,\underline{u}_0) = A(\underline{z}_0,\underline{u}_0); \frac{\partial f}{\partial \underline{u}}(\underline{z}_0,\underline{u}_0) = B(\underline{z}_0,\underline{u}_0) \\ \frac{\partial \varphi}{\partial \underline{z}}(\underline{z}_0,\underline{u}_0) = K_z(\underline{z}_0,\underline{u}_0) \\ \frac{\partial \varphi}{\partial \underline{v}}(\underline{z}_0,\underline{u}_0) = K_v(\underline{z}_0,\underline{u}_0); \frac{\partial h}{\partial \underline{z}}(\underline{z}_0) = C(\underline{z}_0) \end{cases} \qquad (7)$$

the linearized system is then writen :

$$\begin{cases} \dot{\underline{z}}^* = A(\underline{z}_0,\underline{u}_0)\cdot\underline{z}^* + B(\underline{z}_0,\underline{u}_0)\cdot\underline{u}^* \\ \dot{\underline{v}} = -C(\underline{z}_0)\cdot\underline{z}^* = -\underline{w}^* \\ \underline{u}^* = K_z(\underline{z}_0,\underline{u}_0)\cdot\underline{z}^* + K_v(\underline{z}_0,\underline{u}_0)\cdot\underline{v} \end{cases} \qquad (8)$$

K_z and K_v are the gains matrices of the control law by state feedback, functions of the current equilibrium. These gains will be calculated in order to impose on the closed loop system a dynamic evolution compatible with the desired performances. The caracteristic polynomial of the closed-loop process is identified to a damping polynomial chosen in order to impose to the closed-loop system the desired performances. This requires the resolution of a system with $n+q$ nonlinear equations for determining K_z and K_v according to the current equilibrium point $(\underline{z}_0,\underline{u}_0)$. Instead of searching a continuous solution for K_z and K_v, it is possible to discretize the problem in a finished number p of operating points (p = 1...l) and search K_z and K_v for each equilibrium points $(\underline{z}_0^p,\underline{u}_0^p)$. At this stage, the parameters of a control law are known for each predefined equilibrium points. The problem is then the parametric adaptation of the control law according to the current equilibrium which varies when the system evolves. The use of a fuzzy supervisor makes it possible to achieve this objective. The objective is to realize the parametric adaptation of the control law according to the current equilibrium $(\underline{z}_0,\underline{u}_0)$ knowing this in a finished number of operating points $(\underline{z}_0^p,\underline{u}_0^p)$. The input of the decision-making function is then the current equilibrium, and the outputs are the parameters of the control law, that is the gains matrices K_z. The linguistic rule used to reach the aim is then written:

If the curent equilibrium is one of the predefined operating point Then the gains are those calculated for the corresponding predefined equilibrium.

the linked Takagi-Sugeno rules are then writen:

$$\begin{cases} R_p : \text{Si}\left(\underline{z}_0 \text{ est } \underline{\tilde{\mathbf{Z}}}_p\right) \text{ et } \left(\underline{u}_0 \text{ est } \underline{\tilde{\mathbf{U}}}_p\right) \text{ Alors } K = \left[K_z^p \quad K_v^p\right] \quad p = 1...l \\ \underline{z}_0 = \left[z_{10} \quad \cdots \quad z_{n0}\right]^T;\ \underline{\tilde{\mathbf{Z}}}_p = \left[\tilde{\mathbf{z}}_{10}^p \quad \cdots \quad \tilde{\mathbf{z}}_{n0}^p\right]^T \\ \underline{u}_0 = \left[u_{10} \quad \cdots \quad u_{m0}\right]^T;\ \underline{\tilde{\mathbf{U}}}_p = \left[\tilde{\mathbf{u}}_{10}^p \quad \cdots \quad \tilde{\mathbf{u}}_{m0}^p\right]^T \\ K_z^p = \begin{bmatrix} k_{z11}^p & \cdots & k_{z1n}^p \\ \vdots & & \vdots \\ k_{zm1}^p & \cdots & k_{zmn}^p \end{bmatrix};\ K_v^p = \begin{bmatrix} k_{v11}^p & \cdots & k_{v1n}^p \\ \vdots & & \vdots \\ k_{vm1}^p & \cdots & k_{vmn}^p \end{bmatrix} \end{cases} \tag{10}$$

where K is the output of the decision-making function representing the gains of the control law, K_z^p and K_v^p are the matrix gains calculated for the l predefined equilibrium points. The z_{i0}, with $i=1\ldots n$, are the components of $\underline{z}_0$, and u_{j0}, with $j = 1\ldots m$, are the components of $\underline{u}_0$. The $\tilde{\mathbf{z}}_{i0}^p$ (components of $\underline{\tilde{\mathbf{Z}}}_p$) are fuzzy numbers whose modal value z_{i0}^p is defined in order to realize a fuzzy partition of the interval $[(z_{i0}^p)_{min}, (z_{i0}^p)_{max}]$, i.e. :

$$\sum_{p=1}^{l} \mu_{\tilde{\mathbf{z}}_{i0}^p}(z_{i0}) = 1 \quad \forall\ z_{i0} \in \left[(z_{i0}^p)_{min}, (z_{i0}^p)_{max}\right] \tag{11}$$

In this relation $(z_{i0}^p)_{min}$ and $(z_{i0}^p)_{max}$ indicate, respectively largest and smallest value of z_{i0}^p. In the same way, $\tilde{\mathbf{u}}_{j0}^p$ (components of $\underline{\tilde{\mathbf{U}}}_p$) are fuzzy numbers whose modal values u_{j0}^p are defined in order to realize a fuzzy partition of the interval $[(u_{j0}^p)_{min}, (u_{j0}^p)_{max}]$, i.e. :

$$\sum_{p=1}^{l} \mu_{\tilde{\mathbf{u}}_{j0}^p}(u_{j0}) = 1 \quad \forall\ u_{j0} \in \left[(u_{j0}^p)_{min}, (u_{j0}^p)_{max}\right] \tag{12}$$

The gains used in the control law are calculated with the relation (2) which is written, in our case, in the following way:

$$\alpha_p\left(\underline{z}_0(t), \underline{u}_0(t)\right) = \prod_{i=1}^{n} \mu_{\tilde{\mathbf{z}}_{i0}^p}\left(z_{i0}(t)\right) \cdot \prod_{j=1}^{m} \mu_{\tilde{\mathbf{u}}_{j0}^p}\left(u_{j0}(t)\right) \tag{13}$$

$$K_z\left(z_0(t), u_0(t)\right) = \frac{\sum_{p=1}^{l} \alpha_p\left(\underline{z}_0(t), \underline{u}_0(t)\right) \cdot K_z^p}{\sum_{p=1}^{l} \alpha_p\left(\underline{z}_0(t), \underline{u}_0(t)\right)} \tag{14}$$

$$K_v\left(z_0(t), u_0(t)\right) = \frac{\sum_{p=1}^{l} \alpha_p\left(\underline{z}_0(t), \underline{u}_0(t)\right) \cdot K_v^p}{\sum_{p=1}^{l} \alpha_p\left(\underline{z}_0(t), \underline{u}_0(t)\right)} \tag{15}$$

where $(\underline{z}_0(t), \underline{u}_0(t))$ is the evaluation of the equilibrium at time t. The application of this relation thus requires an evaluation of the current equilibrium. This can be carried out, in a continuous way, using the equilibrium conditions, which is not necessarily simple. Another solution is, like previously, to discretize the problem and thus to consider the p operating points $(\underline{z}_0^p, \underline{u}_0^p)$ previously used. The problem is then to estimate the current equilibrium according to a variable to be defined. The output corresponding to the equilibrium $(\underline{z}_0^p, \underline{u}_0^p)$ is $h(\underline{z}_0^p) = \underline{w}_0^p$. The presence of the integrator leads to $\underline{w}_0^p = \underline{r}$. The equilibrium estimation can be then realized from current reference, assuming slow variation which is one of the limitation of this method. However in many applications, the reference step is not applied directly to the process but is used to generate a reference trajectory, which avoid to solicit unnecessarily the actuators. The input of the observation function is thus the current reference and the output the current equilibrium. The linguistic rule used to realize this estimation is:

If the current reference is one the output at the equilibrium Then the current equilibrium is the corresponding predefined equilibrium.

The linked Takagi-Sugeno rules, are then writen:

$$\begin{cases} R_p : \text{If } \underline{r} \text{ is } \underline{\tilde{\mathbf{W}}}_p \text{ Then } \left(\underline{z}_0 = \underline{z}_p\right);\left(\underline{u}_0 = \underline{u}_p\right) \quad p = 1...l \\ \underline{z}_0 = \left[z_{10} \quad \cdots \quad z_{n0}\right]^T; \underline{z}_p = \left[z_{10}^p \quad \cdots \quad z_{n0}^p\right]^T \\ \underline{u}_0 = \left[u_{10} \quad \cdots \quad u_{m0}\right]; \underline{u}_p = \left[u_{10}^p \quad \cdots \quad u_{m0}^p\right]^T \\ \underline{r} = \left[r_1 \quad \cdots \quad r_q\right]^T; \underline{\tilde{\mathbf{W}}}_p = \left[\tilde{\mathbf{w}}_{10}^p \quad \cdots \quad \tilde{\mathbf{w}}_{q0}^p\right]^T \end{cases} \tag{16}$$

In the relation (16), $\underline{r}$ is the input of the observation function representing the current reference. The r_{k0}, with $k = 1\ldots q$, are the components of the reference vector assumed constant or with slow variation. The fuzzy subsets $\tilde{\mathbf{w}}_{k0}^p$ (components of $\underline{\tilde{\mathbf{W}}}_p$) are fuzzy numbers whose modal value w_{k0}^p is defined in order to realize a fuzzy partition of the interval $[(w_{k0}^p)_{min}, (w_{k0}^p)_{max}]$, i.e. :

$$\sum_{p=1}^{l} \mu_{\tilde{\mathbf{w}}_{k0}^p}(w_{k0}) = 1 \quad \forall\ w_{k0} \in \left[(w_{k0}^p)_{min}, (w_{k0}^p)_{max}\right] \tag{17}$$

The estimate of the current equilibrium used in the control law, is obtained using the relation (2):

$$\alpha_p^{'}\left(\underline{r}(t)\right) = \prod_{k=1}^{q} \mu_{\tilde{\mathbf{w}}_{k0}^p}\left(r_i(t)\right) \tag{18}$$

$$z_{i0}\left(r(t)\right) = \frac{\sum_{p=1}^{l} \alpha_p^{'}\left(\underline{r}(t)\right) \cdot z_{i0}^p}{\sum_{p=1}^{l} \alpha_p^{'}\left(\underline{r}(t)\right)};\quad u_{j0}\left(r(t)\right) = \frac{\sum_{p=1}^{l} \alpha_p^{'}\left(\underline{r}(t)\right) \cdot u_{j0}^p}{\sum_{p=1}^{l} \alpha_p^{'}\left(\underline{r}(t)\right)} \tag{19}$$

Where $\underline{r}(t)$ is the reference vector at time t. Notice that the equilibrium thus determined represents, for $\underline{r}$ variable, the equilibrium reached. The system evolves then gradually along the equilibrium trajectories. When $\underline{r}$ is constant, the equality between the reference and the output is ensured by the presence of the integrators. The reference vector r must evolve slowly so that the estimate remains valid. In practice, if the reference are the steps, those will be softened by the introduction of a generator of reference trajectory.

4. APPLICATION OF THE METHOD TO THE CONTROL OF A D.C. MOTOR

The objective is to realize an angular speed feedback with control by the stator current. The motor is assumed fed by a constant armature voltage. This type of control (control by the inductor with free evolution of the rotor current) is the most economic in term of the electronic power because the stator current is very weak in front of the rotor current. The modeling of this system, for a current stator control, leads to a following nonlinear state model:

$$\begin{cases} \dfrac{di_r}{dt} = -\dfrac{R_r}{L_r} \cdot i_r + \dfrac{1}{L_r} \cdot u_r - \dfrac{k \cdot L_s}{L_r} \cdot \omega \cdot i_s \\ \dfrac{d\omega}{dt} = -\dfrac{f}{J} \cdot \omega + \dfrac{k \cdot L_s}{J} \cdot i_r \cdot i_s \end{cases} \tag{20}$$

In the relation (20), k is the constant of couple, R_r rotor resistance, L_r rotor inductance, i_r the rotor current, u_r the rotor voltage maintained constant, ω the rotational speed of the rotor, R_s stator resistance, L_s stator inductance, i_s the stator current which is the control variable of the system. The equilibrium points of the system are given taking equal to zero the derivative of the state vector in (20). The linearized model around an equilibrium point is then given by:

$$\begin{bmatrix} \dot{i}_r^* \\ \dot{\omega}^* \end{bmatrix} = \begin{bmatrix} -\dfrac{R_r}{L_r} & -\dfrac{kL_s i_{s0}}{L_r} \\ \dfrac{kL_s i_{s0}}{J} & -\dfrac{f}{J} \end{bmatrix} \cdot \begin{bmatrix} i_r^* \\ \omega^* \end{bmatrix} + \begin{bmatrix} -\dfrac{kL_s\omega_0}{L_r} \\ \dfrac{kL_s i_{r0}}{J} \end{bmatrix} \cdot i_s^* \tag{22}$$

The control law adopted here is:

$$\begin{cases} \dot{v} = -\omega^* \\ i_s^* = k_1 v + k_2 i_r^* + k_3 \omega^* \end{cases} \tag{23}$$

The caracteristic polynomial of the closed-loop system with a control law (23) is to be identified with the desired caracteristic polynomial allowing to impose desired performances. We now apply the fuzzy supervised control presented above so as to make evolve the system in the speed range 0 to 600 rad/s around 620 rad/s. The stator current varies then in the interval [0.2, 0.4 A]. For the operating point defined by $i_{s0}^1 = 0.4$ A et $i_{s0}^2 = 0.2$ A, the feedback gains are:

$$\begin{aligned} k_1^1 &= -0.00241,\ k_2^1 = 0.00353,\ k_3^1 = 0.00031 \\ k_1^2 &= -0.00247,\ k_2^2 = 0.00186,\ k_3^2 = 0.00084 \end{aligned} \tag{25}$$

The rules base allowing the modification of the feedback gains between these two point are:

$$\begin{cases} R_1 : \text{If } i_{s0} \text{ is } \tilde{i}_{s0}^1 \text{ Then } K = \begin{bmatrix} k_1^1 & k_2^1 & k_3^1 \end{bmatrix} \\ R_2 : \text{If } i_{s0} \text{ is } \tilde{i}_{s0}^2 \text{ Then } K = \begin{bmatrix} k_1^2 & k_2^2 & k_3^2 \end{bmatrix} \end{cases} \tag{26}$$

The fuzzy subset $\tilde{i}_{s0}^1$ et $\tilde{i}_{s0}^2$ are defined on figure 7.

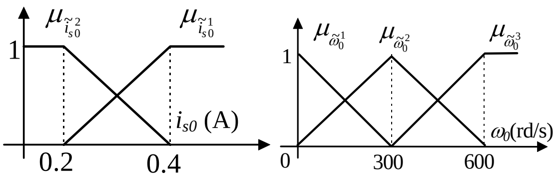

Fig. 7. Definitions of $\tilde{i}_{s0}^1$, $\tilde{i}_{s0}^2$, $\tilde{\omega}_0^1$, $\tilde{\omega}_0^2$, and $\tilde{\omega}_0^3$.

The rules base adopted for the equilibrium estimation are:

$$\begin{cases} R_1' : \text{If } \omega_{0ref} \text{ is } \tilde{\omega}_0^1 \text{ Then } i_{s0}^1 = 0.4;\ \ i_{r0}^1 = 3.876 \\ R_2' : \text{If } \omega_{0ref} \text{ is } \tilde{\omega}_0^2 \text{ Then } i_{s0}^2 = 0.267;\ \ i_{r0}^2 = 8.615 \\ R_3' : \text{If } \omega_{0ref} \text{ is } \tilde{\omega}_0^3 \text{ Then } i_{s0}^3 = 0.2;\ \ i_{r0}^3 = 15.36 \end{cases} \tag{27}$$

wehre ω_{0ref} is the output of the reference generator trajectory. The definitions adopted for the fuzzy subsets $\tilde{\omega}_0^1$, $\tilde{\omega}_0^2$ et $\tilde{\omega}_0^3$ are presented in the figure 8.

4. 1. Generation of a reference trajectory

The objective here is double. The first objective is to generate reference speed slowly variable in order to insure the validity of the presented method. The second objective is to respect the maximum current i_{rmax} admissible by the machine. Indeed, this current can take very important values in transient state if no particular precautions are taken. Note that this objective must be respected whatever the control method used. The generator of reference trajectory corresponding to a constant acceleration is written:

$$\begin{cases} \omega_{ref}'(t) = \gamma t + \omega_0 & \text{si } \ 0 \le t \le T \\ \omega_{ref}'(t) = \omega_{ref} & \text{si } \ t > T \end{cases} \tag{28}$$

where γ represents the acceleration, T is the time necessary for an evolution of the system from initial speed ω_0, value under steady conditions, to a final speed ω_{ref}, defined by the reference. The maximum rotor current i_{rmax} admissible in transient state is 137,5A, the minimal stator current is 0,1A, for a maximum rotational speed of 2500rd/s. This leads to adopt $|\gamma| = 300 rd/s^2$ and $T = 2s$.

4.2. Comparison with a linearizing control

The linearizing control is a nonlinear control technic based on a change of coordinates allowing to linearize the system (Isidori 1989, Isidori 1995). The application of this method to our system leads to the following linearizing control law:

$$i_s(t) = \frac{\left[J\left(k_0 \int_{t0}^{t} \left(r(\tau) - \omega(\tau) \right) d\tau + k_1 \omega(t) \right) + f\omega(t) \right] k L_s i_r(t)}{J^2} \quad (29)$$

where the parameters k_0 and k_1 are determined in manner to obtain the desired dynamic performances. By adopting identical performances to those defined by (24), we obtain: $k_0 = 13{,}2;\ k_1 = 3{,}6$. The recordings presented on figures 9 and 10 compare, for various amplitudes of the step reference, the results obtained for a supervised and linearizing control law. The step reference is not applied directly to the system, but is used to generate the reference trajectory in accordance with the relations (28), in order to not exceed the maximum rotor current admissible by the machine. According to figures 9 and 10, it is clear that the dynamic performances of the supervised system are preserved, which is also the case, obviously, with the linearizing control. The performances obtained by the fuzzy supervision technique are equivalent to the linearisante control.

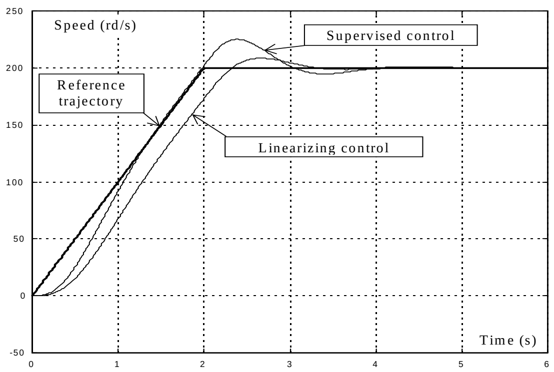

Fig. 9. Response to a step reference of 200 rad/s.

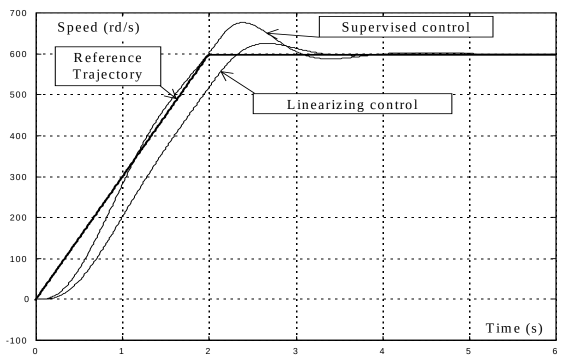

Fig. 10. Response to a step reference of 600 rad/s.

5. CONCLUSION

The presented method allows the invariance of dynamic performance of the closed-loop system in all operating range. The problems of the parametric adaptation of the control law and the equilibrium estimation are solved by the use of a fuzzy supervisor containing Takagi-Sugeno rules base type. The synthesis of the fuzzy supervisor rests on the knowledge of the gains and the equilibrium for a finished number of operating point, which constitutes a discretisation. This discretization is very important from a practical point of view. Indeed this method does not require necessarily the knowledge of a global model, generally nonlinear, but is satisfied with a finished number of local linear models. This method was applied to the control by the inductor of a D.C. motor. A generator of reference trajectory is installed in order to limit the points of current and to ensure a more progressive variation of the reference. The performances obtained with the supervised control were compared with the linearizing control of Isidori. The results are satisfactory and lead to the validation of this method.

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