# CONTINUOUS NONLINEAR ADAPTATIVE CONTROL OF AN ACTIVE VEHICLE SUSPENSION

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ABSTRACT: The aim of this paper is to present the control of a conventional suspension with an electromagnetic actuator. The principle of the control rests on the compensation of the non linearity so as to obtain a suspension whose stiffness and damping are programmable. The main difficulty of this approach is the ignorance of the static characteristics. This difficulty is solved by using a continuous identification of the nonlinear functions characterizing the components of the suspension. The approximation functions used are the Fuzzy Inference Systems (FIS) which have the property of universal approximator. According to this principle, the control developed in this work leads to a new continuous nonlinear adaptive control algorithm. 

KEYWORDS: Electric actuator, Approximation function, Universal approximator, Fuzzy Inference Systems, Nonlinear adaptative control, Active vehicle suspension.

## 1. INTRODUCTION

A car suspension is meant to absorb vibrations caused by the profile of the road in order to improve the riding comfort. The road-holding quality is another requirement which must also be taken into account as far as safety is concerned. Both demands are conflicting i.e. we talk of the dilemme over riding comfort versus road-holding. A classical suspension uses a spring which allows the contact of the tyre with the ground in spite of any disturbance which might occur during the drive. However the spring alone is not sifficient as such a device implies natural oscillatory movements. It is thus necessary to introduce a viscous friction which very quickly absorbs the oscillations that is the damper.

The traditional suspension used on motor vehicles consists of the two following basic components : the spring and the damper, the mechanical characteristics of which are not adjustable (the stiffness and the friction coefficient). These two parts are dimensioned in order to reach the best possible arrangement between the riding comfort and the road-holding qualities in a wide range of operating conditions. However, so many different road profiles and loading conditions are involved that they do not allow an optimal behavior in any situation owing to the passivity of the elements.

Many works [Buc. 01], [Dun. 91], [Red. 89], [Yas. 91], [Yue. 89], showed that introducing active parts in the suspension leads to a remarkable improvement in the road handling in spite of the varying operating conditions. Hydraulic components are used on the present active suspension. They are controlled in order to obtain an ajustable stiffness and damping. It is then possible to adjust the characteristics of the suspension to fit the different loading conditions and the profile of the road. Thus we obtain a compromise between the riding comfort and the road-holding qualities which is satisfying in all circumstances. However, the control law used for the conventional active vehicle suspension does not take into account the non linearity of the stiffness and damping characteristics. This work focuses on a nonlinear adaptative control of an active vehicle suspension.

The following work proposes to use an electric actuator for the realization of an active suspension. This actuator compared to an hydraulic one offers a much greater robustness and needs almost no maintenance. The specificity of the application considered lead us to design an original actuator which has been patented. This paper is divided into four parts. In the first part, after having described the general principle of the actuator used, we present a modelling intended for the synthesis of a control in effort of the actuator. In the second part, we are interested in the active suspension of a motor vehicle for which the actuator proposed could be used. In this second part we propose a control law allowing to obtain an ajustable stiffness and damping. However, the synthesis of this control law requires the characteristics of stiffness and damping which are unfortunately non-linear and imperfectly known. To overcome this difficulty, we propose in the third part a continuous nonlinear adaptive control. The functions of approximation used for the estimation of nonlinear characteristics are fuzzy inferences systems (FIS). We finally present a proof of the stability of the proposed control law. Various simulation and test results intended to validate the principles of the suggested control law are presented in the fourth part.

## 2. WORKING PRINCIPLE, MODELLING AND EFFORT CONTROL OF THE ACTUATOR

Figure 1 presents the schematic diagram of the proposed actuator. It consists of a mobile cylindrical core, as well as bolt wich makes it possible to channel the lines of a field and it also ensure the mechanical rigidity of the unit and a winding intended to create a magnetic field.

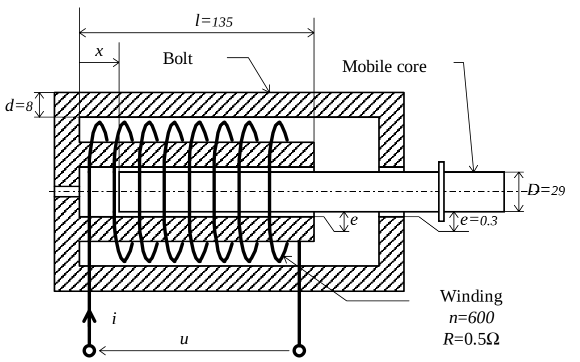



Fig. 1. Schematic diagram of the actuator.

The origin of the force being exerted on the core under the action of the magnetic field created by the winding, comes from the presence of the gap *e* between the core and the bolt. The moving part of the system moves in the direction of a minimal magnetic reluctance, i.e. in the direction of an increase in surfaces in glances. Please note that this actuator cannot exert itself on both directions. Thus, two actuators must be considered, one used for pushing and the other for pulling.

### *2.1. Modelling of the actuator*

The winding of the actuator, supplied under a potential difference $u$, is traversed by an electrical current $i$ giving rise to a magnetic field whose lines of fields are channeled by the bolt and the mobile core. Under the action of this field, the ferromagnetic core is subjected to a magnetic force $F_m$ function of position $x$ of the core and current (figure 1). The expression of the force can be given by calculating the energy stored in the system, this one is expressed by the following relation:

$$E(\phi, x) = \int_0^{\phi} \mathcal{R}(x)\phi d\phi = \frac{1}{2}\mathcal{R}(x)\phi^2 \qquad (1)$$

where $\mathcal{R}$ represents the reluctance magnetic circuit which varies with the displacement of the core and $\phi$ the magnetic flux. The application of the Ampère theorem leads to the well-known Hopkinson law connecting the magnetomotive force $\mathcal{F}$ to the reluctance and the magnetic flux: $\mathcal{F} = ni = \mathcal{R}(x)\phi$. Energy is then written:

$$E(\phi, x) = \frac{(ni)^2}{2\mathcal{R}(x)} \qquad (2)$$

The power provided to the system appears as the sum of the electric power and the mechanical power:

$$P = \frac{dE}{dt} = \frac{\partial E}{\partial \phi}\frac{d\phi}{dt} + \frac{\partial E}{\partial x}\frac{dx}{dt} = ui + F_m v \qquad (3)$$

The magnetic force is thus written:

$$F_m = \frac{\partial E}{\partial x} = \frac{1}{2}(ni)^2 \frac{d}{dx}\left(\frac{1}{\mathcal{R}(x)}\right) \qquad (4)$$

This last relation suggests the possibility of using a model of the form:

$$F_m(i, x) = p(x)i^2 \qquad (5)$$

where $p(x)$ is a polynomial, the coefficients of which are given starting from experimental results; it then leads to the following relation:

$$F_m(i, x) = \left(0.197 - 1.94x + 26.34x^2 - 239.54x^3\right) \cdot i^2 \qquad (6)$$

Figure 2 presents the law of effort according to the position obtained using the relation (6) compared with the experimental results.

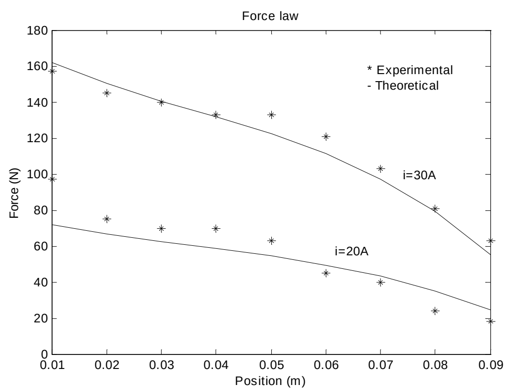



Fig. 2. *Theoretical and experimental results of the force law.*

Figure 3 presents the Bond-Graph model of the actuator where $m$ is the total mass of the moving part and $c$ the coefficient of viscous friction present in the guidance of the core.

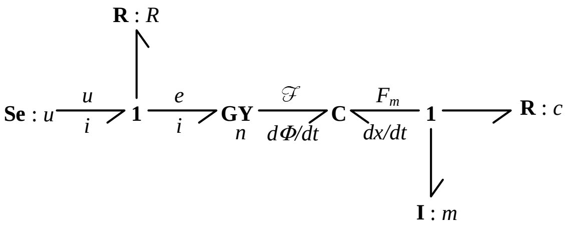



Fig. 3. *Bond-Graph model of the actuator.*

Taking into account this representation, the state space model of the actuator is given by the system (7).

$$\begin{cases} \dfrac{dx}{dt} = v \\ \dfrac{dv}{dt} = -\dfrac{c}{m}v - \dfrac{1}{m}\,p(x)i^2 \\ \dfrac{di}{dt} = -\dfrac{R}{L}i + \dfrac{1}{L}u \end{cases} \tag{7}$$

*2.2. Effort control of the actuator*

The objective is to carry out a control in effort of the system of figure 1. According to the relation (5), the reference current of $i_r$ to impose on the system so that it produces the desired effort $F_m$ writes $i_r = \sqrt{F_m / p(x)}$. The electromagnetic force $F_m$ thus can be obtained by controlling the current in the actuator with the reference current of $i_r$ using the following control law:

$$u(t) = \frac{1}{Ti}\int_0^t \sqrt{\frac{F_m(\tau)}{p(x)}} - i(\tau)d\tau - K_i i(t) \tag{8}$$

The functional diagram of figure 4 presents the control of the actuator making it possible to obtain the desired effort $F_m$ .

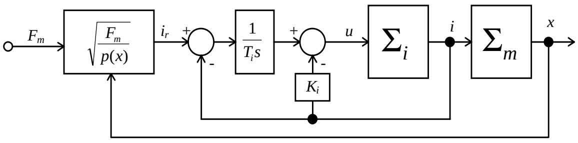



Figure 4. *Effort control of the actuator..*

Blocks $\Sigma_i$ and $\Sigma_m$ respectively represent the dynamics of the electric part and the mechanical part of the actuator. The suggested control law in addition to the linearization of the system makes it possible to impose the desired effort $F_m$. The electric part is controlled in current using a conventional proportional and integral compensator. The reference current is calculated according to the desired force.

## 3. APPLICATION TO THE ACTIVE SUSPENSION

Figure 5 presents the mechanical structure of an active suspension as well as the corresponding Bond-Graph model. $M$ is the body mass, $k_M$ the spring coefficient of suspension, $b_M$ the damper coefficient of suspension, $F_m$ represents the effort developed by the actuator considered above, $m$ the wheel mass and $k_m$ the stiffness of the tyre.

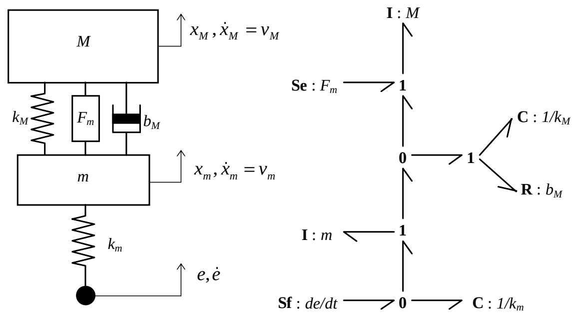



Fig. 5. *Mechanical structure of an active suspension and its corresponding Bond-Graph model.*

If we take into account the Bond-Graph model of figure 5, the state space representation of the system is written as follows:

$$\begin{cases} \dfrac{dx_M}{dt} = v_M \\ \dfrac{dx_m}{dt} = v_m \\ \dfrac{dv_M}{dt} = \dfrac{1}{M}\left[k_M(x_m - x_M) + b_M(v_m - v_M) + F_m\right] \\ \dfrac{dv_m}{dt} = -\dfrac{1}{m}\left[k_M(x_m - x_M) + b_M(v_m - v_M) - k_m(e - x_m) + F_m\right] \end{cases} \tag{9}$$

The stiffness and the damping of a conventional suspension are respectively nonlinear functions of the position variation $\Delta_x = x_m - x_M$ and speed variation $\Delta_v = v_m - v_M$, which we will write: $k_M = f_k(\Delta_x)$ and $b_M = f_b(\Delta_v)$ [Buc. 01]. The state space representation is then:

$$\begin{cases} \dfrac{dx_M}{dt} = v_M \\ \dfrac{dx_m}{dt} = v_m \\ \dfrac{dv_M}{dt} = \dfrac{1}{M}\left[f_k(\Delta_x)\Delta_x + f_b(\Delta_v)\Delta_v + F_m\right] \\ \dfrac{dv_m}{dt} = -\dfrac{1}{m}\left[f_k(\Delta_x)\Delta_x + f_b(\Delta_v)\Delta_v - k_m(e - x_m) + F_m\right] \end{cases} \tag{10}$$

The functions $f_k(\Delta_x)$ and $f_b(\Delta_v)$ being nonlinear, the dynamic behavior of the suspension varies according to the operating conditions, which can be prejudicial to the comfort and the handling in case of severe conditions of operation. The principle of the control developed in the continuation consists in the compensation of the non linearities of the suspension, so as to obtain a dynamic behavior which is independent of the operating conditions.

Let , respectively, $k_M^r$ and $b_M^r$ be the desired stiffness and damping, the problem is to build a control law so that the suspension behaves like the following reference model:

$$\begin{cases} \frac{dx_M}{dt} = v_M \\ \frac{dx_m}{dt} = v_m \\ \frac{dv_M}{dt} = \frac{1}{M}\left[k_M^r \Delta_x + b_M^r \Delta_v\right] \\ \frac{dv_m}{dt} = -\frac{1}{m}\left[k_M^r \Delta_x + b_M^r \Delta_v - k_m (e - x_m)\right] \end{cases} \tag{11}$$

Comparing (10) and (11), the required control law is written:

$$F_m = \left(k_M^r - f_k(\Delta_x)\right)\Delta_x + \left(b_M^r - f_b(\Delta_v)\right)\Delta_v \tag{12}$$

This control law requires the knowledge of the nonlinear functions $f_k(\Delta_x)$ and $f_b(\Delta_v)$, which is not usually the case. We then propose to build the control law (12) from a real time estimation of the unknown functions [Cal. 01], [San. 92], [Tzi. 92].

## 4. NONLINEAR ADAPTIVE CONTROL

In this part one proposes to seek estimators $\hat{f}_k(\Delta_x, \underline{\theta}_k)$ and $\hat{f}_b(\Delta_v, \underline{\theta}_v)$ allowing to approach the unknown functions $f_k(\Delta_x)$ and $f_b(\Delta_v)$, where the parameters vectors $\underline{\theta}_k$ and $\underline{\theta}_v$ are adjusted so that the following quadratic criteria are minimized.

$$\begin{cases} J_k(\underline{\theta}_k) = \frac{1}{2T}\int_0^T \left(f_k(\Delta_x) - \hat{f}_k(\Delta_x, \underline{\theta}_k)\right)^2 dt \\ J_b(\underline{\theta}_v) = \frac{1}{2T}\int_0^T \left(f_b(\Delta_v) - \hat{f}_b(\Delta_v, \underline{\theta}_v)\right)^2 dt \end{cases} \tag{13}$$

where $T$ represents the horizon on which optimization is carried out.

### *4.1. Fuzzy Inferences Systems like approximation functions*

The choice of the functions $\hat{f}_k(\Delta_x, \underline{\theta}_k)$ and $\hat{f}_b(\Delta_v, \underline{\theta}_v)$ is not indifferent. These must have the property of universal approximator in order to ensure the existence of solutions. We propose to build these functions using fuzzy inferences systems (FIS). This choice is justified by their capacities of approximation and by their easility to integrate qualitative knowledge under the shapes of linguistic rules. Indeed, it was shown that the FIS are universal approximators [Wan. 92], in other words any sufficiently regular function can be approximated with an arbitrary precision, by a FIS, after a suitable adjustment of the parameters using a training algorithm.

Note that the neural networks [Hor. 94], polynomial functions or Fourier series for example have also this property. However, the enormous advantage of the FIS comparatively to the other approximators is their aptitudes to integrate qualitative knowledge which expressed by using linguistic rules, fixes the structure of the network (structural tuning). From this point of view, we can then speak of a linguistic knowledge model, whereas the other approximators are rather purely numerical models. The fact that a SIF defines implicitly a nonlinear function makes it possible to use the existing techniques of parametric adaptation of the usual approximator to carry out the tuning of the FIS.

A monovariable Fuzzy Inference System of input $u$ and output $y$ is a nonlinear function $y = \varphi(u, \underline{\theta})$ defined by the following system:

$$\begin{cases} y = \Delta\left(\Phi(u) \circ \tilde{\mathcal{R}}\right) \\ \tilde{\mathcal{R}} = \bigcup_i \tilde{\boldsymbol{A}}_i \times \tilde{\boldsymbol{B}}_i, \quad i = 1 \ldots N \end{cases} \tag{14}$$

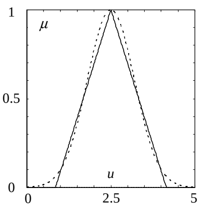
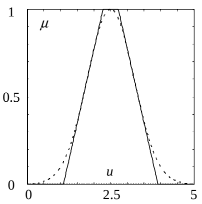
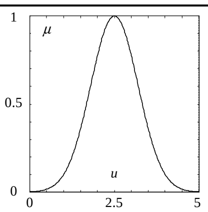
where $\Phi$ is an operator of fuzzification, $\tilde{\mathcal{R}}$ is a fuzzy relation making it possible to modelize the rules base used to build the function $\varphi$, $N$ represents the total number of rules. The symbol ° is an operator of fuzzy inference making it possible to calculate the image of the fuzzy subset resulting from the fuzzification, by the fuzzy relation $\tilde{\mathcal{R}}$. $\Delta$ is an operator of defuzzification making it possible to transform the result of the inference which is a fuzzy subset in a numerical value directly usable for the control. $\tilde{\boldsymbol{A}}_i$ and $\tilde{\boldsymbol{B}}_i$ are fuzzy subsets defined, respectively, by the following membership functions $\mu_{\tilde{\mathbf{A}}_i}(u, \underline{\alpha}_i)$, $\mu_{\tilde{\mathbf{B}}_i}(y, \underline{\beta}_i)$. These functions are parameterized by the vectors $\underline{\alpha}_i$, $\underline{\beta}_i$ with $i = 1 \ldots N$ which are the adjustable elements of the FIS. In the continuation we consider the case where the $\tilde{\boldsymbol{B}}_i$ are defined by fuzzy singletons and the $\tilde{\boldsymbol{A}}_i$ by triangular, trapezoidal, or Gaussian membership functions. Table 1 gives the corresponding expressions of these different membership functions.

Table 1 Various membership functions used.

| | |
|---|---|
| Singleton | $\mu_{\tilde{B}_i}(y,\beta_i)=\begin{cases}1\, if \;\; y=\beta_i\\ 0\end{cases}$ |
| Triangular | $\begin{cases}\mu_{\tilde{A}_i}(x,\underline{\alpha}_i)=\max\left(\min\left(\frac{x-\alpha_1^i}{\alpha_2^i-\alpha_1^i},\frac{\alpha_3^i-x}{\alpha_3^i-\alpha_2^i}\right),0\right)\\ \underline{\alpha}_i=\left[\alpha_1^i \quad \alpha_2^i \quad \alpha_3^i\right]^T\end{cases}$ |
| Trapézoidal | $\begin{cases}\mu_{\tilde{A}_i}(x,\underline{\alpha}_i)=\max\left(\min\left(\frac{x-\alpha_1^i}{\alpha_2^i-\alpha_1^i},1,\frac{\alpha_4^i-x}{\alpha_4^i-\alpha_3^i}\right),0\right)\\ \underline{\alpha}_i=\left[\alpha_1^i \quad \alpha_2^i \quad \alpha_3^i \quad \alpha_4^i\right]^T\end{cases}$ |
| Gaussian | $\begin{cases}\mu_{\tilde{A}_i}(x,\underline{\alpha}_i)=\exp\left[-\frac{1}{2}\left(\frac{x-\alpha_1^i}{\alpha_2^i}\right)^2\right]\\ \underline{\alpha}_i=\left[\alpha_1^i \quad \alpha_2^i\right]^T\end{cases}$ |

In order to limit the number of parameters, we propose a formulation using in all cases only two parameters of adjustment (table 2).

Table 2 Parameter setting of the membership functions.

| | |
|---|---|
| Triangular membership function<br>$\begin{cases}\mu_{\tilde{A}_i}(u,m_i,\sigma_i)=\max\left(\min\left(\alpha_1^i,\alpha_2^i\right),0\right)\\ \alpha_1^i=\frac{\sqrt{2\ln(2)}(u-m_i)+4\sigma_i\ln(2)}{4\sigma_i\ln(2)}\\ \alpha_2^i=\frac{\sqrt{2\ln(2)}(m_i-u)+4\sigma_i\ln(2)}{4\sigma_i\ln(2)}\end{cases}$ |  |
| Trapezoidal membership function<br>$\begin{cases}\mu_{\tilde{A}_i}(u,m_i,\sigma_i)=\max\left(\min\left(\alpha_1^i,1,\alpha_2^i\right),0\right)\\ \alpha_1^i=\frac{\sqrt{2\ln(2)}(u-m_i)+\sigma_i(1+2\ln(2))}{2\sigma_i}\\ \alpha_2^i=\frac{\sqrt{2\ln(2)}(m_i-u)+\sigma_i(1+2\ln(2))}{2\sigma_i}\end{cases}$ |  |
| Gaussian membership function<br>$\mu_{\tilde{A}_i}(u,m_i,\sigma_i)=\exp\left(-\frac{1}{2}\left(\frac{x-m_i}{\sigma_i}\right)^2\right)$ |  |

If the fuzzification is realised by singleton and the fuzzy inference operator is the supremum-product and the defuzzification is carried out by the centre of gravity, one shows that the analytical expressions of the FIS (14) is written [Lee. 90], [Dri. 96]:

$$\begin{cases}y=\varphi(u,\underline{\theta})=\sum_{i=1}^{N}\beta_i\psi_i(u,\underline{\alpha}_i)\\ \text{with} \quad \psi_i(u,\underline{\alpha}_i)=\dfrac{\mu_{\tilde{A}_i}(u,\underline{\alpha}_i)}{\sum_{i=1}^{N}\mu_{\tilde{A}_i}(u,\underline{\alpha}_i)}\\ \underline{\theta}=\left[\beta_1 \quad \cdots \quad \beta_N \quad \underline{\alpha}_1 \quad \cdots \quad \underline{\alpha}_N\right]^T\\ \text{with} \quad \underline{\alpha}_i=\left[m_i \quad \sigma_i\right]^T, \quad i=1\ldots N\end{cases} \tag{15}$$

A FIS defined such as (15) makes it possible to approach any continuous function on a given compact of the variables with an arbitrary precision related to the number of rule. An important difficulty is to adjust the parameters vector $\underline{\theta}$ so as to carry out a good approximation of the unknown functions. This problem can be simplified by adopting a regular distribution of the membership functions on the interval of evolution of the input variable *u*. In the case of a uniform distribution of the membership functions, we obtain for the parameter vectors $\underline{\alpha}_i$:

$$\begin{cases}m_i=(i-1)\dfrac{u_{\max}-u_{\min}}{N-1}+u_{\min}, \quad i=1,\ldots,N\\ \sigma_i=\sigma=\dfrac{u_{\max}-u_{\min}}{2\sqrt{2\ln(2)}(N-1)}, \quad i=1,\ldots,N\\ u\in\left[u_{\min} \quad u_{\max}\right]\end{cases} \tag{16}$$

*4.2. The certainty equivalence assumption*

Taking into account (15) and (16), we suppose that for $N_k$ and $N_v$ given, there is an interval of the variables $\Delta_x\in\left[(\Delta_x)_{\min},(\Delta_x)_{\max}\right]$ and $\Delta_v\in\left[(\Delta_v)_{\min},(\Delta_v)_{\max}\right]$ on which the unknown and continuous nonlinear functions $f_k(\Delta_x)$ and $f_b(\Delta_v)$ can be expressed using the following relations:

$$\begin{cases}f_k(\Delta_x)=\sum_{i=1}^{N_k}\beta_k^i\psi_k^i(\Delta_x)\\ f_b(\Delta_v)=\sum_{i=1}^{N_b}\beta_b^i\psi_b^i(\Delta_v)\end{cases} \tag{17}$$

where the basic function $\psi_k^i(\Delta_x)$ and $\psi_b^i(\Delta_v)$ are parameterized in accordance with (15) and (16), and where the parameters $\beta_k^i$ and $\beta_b^i$ are unknown. Taking into account (17) the selected approximations functions are:

$$\begin{cases}\hat{f}_k\left(\Delta_x,\underline{\hat{\beta}}_k\right)=\sum_{i=1}^{N_k}\hat{\beta}_k^i\psi_k^i(\Delta_x)\\ \hat{f}_b\left(\Delta_v,\underline{\hat{\beta}}_b\right)=\sum_{i=1}^{N_b}\hat{\beta}_b^i\psi_b^i(\Delta_v)\\ \underline{\hat{\beta}}_k=\left[\hat{\beta}_k^1 \quad \cdots \quad \hat{\beta}_k^{N_k}\right]^T,\underline{\hat{\beta}}_b=\left[\hat{\beta}_b^1 \quad \cdots \quad \hat{\beta}_b^{N_b}\right]^T\end{cases} \tag{18}$$

where $\hat{\beta}_k^i$ and $\hat{\beta}_b^i$ are estimates of the unknown parameters $\beta_k^i$ and $\beta_b^i$. Let us note that the values provided by the unknown functions $f_k(\Delta_x)$ and $f_b(\Delta_v)$ are not directly measurable, it is then necessary to build, from the measurable variables $\Delta_x$ and $\Delta_v$, a measurement of the estimation error from which the adjustment will be carried out.

*4.3.The continuous law of the parameters adjustment*

According to (10), we have:

$$\dot{v}_M = \frac{1}{M}\left[ f_k(\Delta_x)\Delta_x + f_b(\Delta_v)\Delta_v + F_m \right] \tag{19}$$

one can then build the following estimator:

$$\dot{\hat{v}}_M = -\alpha(\hat{v}_M - v_M) + \frac{1}{M}\left[\hat{f}_k(\Delta_x)\Delta_x + \hat{f}_b(\Delta_v)\Delta_v + F_m\right] \tag{20}$$

The dynamics of the estimation error $\tilde{v}_M = v_M - \hat{v}_M$ is then written:

$$\begin{aligned}\dot{\tilde{v}}_M &= \dot{v}_M - \dot{\hat{v}}_M = \alpha(\hat{v}_M - v_M) \\ &+ \frac{1}{M}\left[\left(f_k(\Delta_x) - \hat{f}_k\left(\Delta_x, \underline{\hat{\beta}}_k\right)\right)\Delta_x + \left(f_b(\Delta_v) - \hat{f}_b\left(\Delta_v, \underline{\hat{\beta}}_b\right)\right)\Delta_v\right]\end{aligned} \tag{21}$$

The problem is to seek the parameter vector of the functions $\hat{f}_k\left(\Delta_x, \underline{\hat{\beta}}_k\right)$ and $\hat{f}_b\left(\Delta_v, \underline{\hat{\beta}}_b\right)$ so as to ensure the convergence to zero of the estimation error: $\lim_{t\to+\infty} \tilde{v}_M(t) = 0$. For this purpose one can seek the law of adjustment of the parameters allowing the minimization of the following quadratic criterion:

$$\begin{cases} J\left(\underline{\hat{\beta}}\right) = \dfrac{1}{2T}\displaystyle\int_0^T \tilde{v}_M^2\left(t, \underline{\hat{\beta}}\right)dt \\ \underline{\hat{\beta}} = \begin{bmatrix}\hat{\beta}_k^1 & \cdots & \hat{\beta}_k^{N_k} & \hat{\beta}_b^1 & \cdots & \hat{\beta}_b^{N_b}\end{bmatrix}^T \end{cases} \tag{22}$$

The law of adjustment of the parameters must be such as the criterion is decreasing in the time, it is thus necessary that $\dot{J}\left(\underline{\hat{\beta}}\right) < 0$, that is to say still too $\dot{J}\left(\underline{\hat{\beta}}\right) = \left(\partial J / \partial \underline{\hat{\beta}}\right)^T \dot{\underline{\hat{\beta}}} < 0$. This last inequality is hold if:

$$\dot{\underline{\hat{\beta}}} = -\lambda \frac{\partial J}{\partial \underline{\hat{\beta}}} \qquad 23)$$

that is to say that the parameter vector evolves in an opposite direction to the gradient of the criterion. This is the well-known gradient method. In case of a continuous adaptation of parameters (i.e. $J = \tilde{v}_M^2/2$), one obtains the following relations:

$$\begin{cases} \dot{\hat{\beta}}_k^i(t) = -\lambda_k \dfrac{\partial \tilde{v}_M}{\partial \hat{\beta}_k^i}\tilde{v}_M, \quad i = 1, \ldots, N_k \\ \dot{\hat{\beta}}_b^i(t) = -\lambda_b \dfrac{\partial \tilde{v}_M}{\partial \hat{\beta}_b^i}\tilde{v}_M, \quad i = 1, \ldots, N_b \end{cases} \tag{24}$$

where $\lambda_k$ and $\lambda_v$ are coefficients making it possible to accelerate the convergence of the estimation. The calculation of $\partial\tilde{v}_M / \partial\hat{\beta}_k^i$ requires the analytical expression of $\tilde{v}_M$ i.e. the solution of the equation (21). However $\tilde{v}_M$ represents the filtering of the quantity $\left(\left(f_k(\Delta_x) - \hat{f}_k\left(\Delta_x, \underline{\hat{\beta}}_k\right)\right)\Delta_x + \left(f_b(\Delta_v) - \hat{f}_b\left(\Delta_v, \underline{\hat{\beta}}_b\right)\right)\Delta_v\right)/M$, if the time-constant $1/\alpha$ of the filter is selected sufficiently small one can reasonably admit that:

$$\tilde{v}_M \approx \frac{1}{M}\left[\left(f_k(\Delta_x) - \hat{f}_k\left(\Delta_x, \underline{\hat{\beta}}_k\right)\right)\Delta_x + \left(f_b(\Delta_v) - \hat{f}_b\left(\Delta_v, \underline{\hat{\beta}}_b\right)\right)\Delta_v\right] \tag{25}$$

Taking into account (18), the relation (25) is written:

$$\tilde{v}_M \approx \frac{1}{M}\left[\left(f_k(\Delta_x) - \sum_{i=1}^{N_k}\hat{\beta}_k^i\psi_k^i(\Delta_x)\right)\Delta_x + \left(f_b(\Delta_v) - \sum_{i=1}^{N_b}\hat{\beta}_b^i\psi_b^i(\Delta_v)\right)\Delta_v\right] \tag{26}$$

from where in accordance with (24) the law of adjustment of the parameters is the following one:

$$\begin{cases} \dot{\hat{\beta}}_k^i(t) = \dfrac{\lambda_k}{M}\psi_k^i(\Delta_x)\Delta_x\tilde{v}_M, \quad i = 1, \ldots, N_k \\ \dot{\hat{\beta}}_b^i(t) = \dfrac{\lambda_b}{M}\psi_b^i(\Delta_v)\Delta_v\tilde{v}_M, \quad i = 1, \ldots, N_b \end{cases} \tag{27}$$

The control law (12) making it possible to have a suspension of stiffness $k_M^r$ and damping $b_M^r$ is then written:

$$F_m = \left(k_M^r - \sum_{i=1}^{N_k}\hat{\beta}_k^i\psi_k^i(\Delta_x)\right)\Delta_x + \left(b_M^r - \sum_{i=1}^{N_b}\hat{\beta}_b^i\psi_b^i(\Delta_v)\right)\Delta_v \tag{28}$$

which corresponds to a nonlinear adaptive control by state feedback.

*4.4. Stability*

The application of the continuous adjustment law (27) leads to an asymptotic convergence to zero of the estimation error, consequently the closed loop system behaves like a suspension of stiffness $k_M^r$ and damping $b_M^r$. To show it let us consider the following candidate Lyapunov function:

$$V = \frac{1}{2}\left(\tilde{v}_M^2 + \frac{1}{\lambda_k}\sum_{i=1}^{N_k}\left(\tilde{\beta}_k^i\right)^2 + \frac{1}{\lambda_b}\sum_{i=1}^{N_k}\left(\tilde{\beta}_b^i\right)^2\right) \tag{29}$$

where $\tilde{\beta}_k^i = \beta_k^i - \hat{\beta}_k^i$ and $\tilde{\beta}_b^i = \beta_b^i - \hat{\beta}_b^i$ are the estimation errors of the unknown parameters respectively $\beta_k^i$ and $\beta_b^i$. The time derivative of $V$ is written:

$$\dot{V} = \dot{\tilde{v}}_M \tilde{v}_M - \frac{1}{\lambda_k} \sum_{i=1}^{N_k} \dot{\hat{\beta}}_k^i \tilde{\beta}_k^i - \frac{1}{\lambda_b} \sum_{i=1}^{N_k} \dot{\hat{\beta}}_b^i \tilde{\beta}_b^i \qquad (30)$$

replacing $\dot{\tilde{v}}_M$, $\dot{\hat{\beta}}_k^i$ and $\dot{\hat{\beta}}_b^i$ by their expressions (21), (27) and taking into account (17) and (18), one obtains:

$$\begin{aligned} \dot{V} = & -\alpha\tilde{v}_M^2 + \frac{1}{M}\left[\sum_{i=1}^{N_k} \tilde{\beta}_k^i \psi_k^i(\Delta_x)\Delta_x + \sum_{i=1}^{N_k} \tilde{\beta}_b^i \psi_b^i(\Delta_v)\Delta_v\right]\tilde{v}_M \\ & -\frac{1}{\lambda_k}\sum_{i=1}^{N_k} \frac{\lambda_k}{M}\psi_k^i(\Delta_x)\Delta_x \tilde{v}_M \tilde{\beta}_k^i \\ & -\frac{1}{\lambda_b}\sum_{i=1}^{N_k} \frac{\lambda_b}{M}\psi_b^i(\Delta_v)\Delta_v \tilde{v}_M \tilde{\beta}_b^i \end{aligned} \qquad (31)$$

finally one have $\dot{V} = -\alpha\tilde{v}_M^2$ which is strictly negative, then the estimation error converges to zero.

## 5. SIMULATION RESULTS

In this part the control law (28) with the mechanism of adjustment (24) is applied to the suspension described figure 5 the nominal parameters of which are the following: $M$=290Kg, $m$=28.5Kg, $k_m$=155900N/m, $k_M$=15000N/m, $b_M$=1500N/m/s. The rules base used for the approximators $\hat{f}_k$ and $\hat{f}_b$ is the following one:

$$\begin{cases} \text{If } \Delta_x \text{ is } \tilde{\boldsymbol{A}}_k \text{ then } \hat{f}_k = \hat{\beta}_k^1 \\ \text{If } \Delta_x \text{ is not}\left(\tilde{\boldsymbol{A}}_k\right) \text{then } \hat{f}_k = \hat{\beta}_k^2 \\ \text{If } \Delta_v \text{ is } \tilde{\boldsymbol{A}}_b \text{ then } \hat{f}_b = \hat{\beta}_b^1 \\ \text{If } \Delta_v \text{ is not}\left(\tilde{\boldsymbol{A}}_b\right) \text{then } \hat{f}_b = \hat{\beta}_b^2 \\ \Delta_x \in [-0.2, 0.2]; \quad \Delta_v \in [-20, 20] \end{cases} \qquad (32)$$

Defining the fuzzy subsets $\tilde{\boldsymbol{A}}_k$ and $\tilde{\boldsymbol{A}}_b$ by triangular memberships function and taking into account the parameter setting (16), one obtains the following expressions of the approximators $\hat{f}_k$ and $\hat{f}_b$:

$$\begin{cases} \hat{f}_k\left(\Delta_x, \underline{\hat{\beta}}_k\right) = \hat{\beta}_k^1 \max\left(\min\left(\alpha_k\Delta_x + 1, 1 - \alpha_k\Delta_x\right), 0\right) \\ + \hat{\beta}_k^2\left[1 - \max\left(\min\left(\alpha_k\Delta_x + 1, 1 - \alpha_k\Delta_x\right), 0\right)\right] \\ \hat{f}_b\left(\Delta_v, \underline{\hat{\beta}}_b\right) = \hat{\beta}_b^1 \max\left(\min\left(\alpha_b\Delta_v + 1, 1 - \alpha_b\Delta_v\right), 0\right) \\ + \hat{\beta}_b^2\left[1 - \max\left(\min\left(\alpha_b\Delta_v + 1, 1 - \alpha_b\Delta_v\right), 0\right)\right] \\ \alpha_k = \dfrac{50\sqrt{2}}{17\sqrt{\ln(2)}}, \quad \alpha_b = \dfrac{\sqrt{2}}{34\sqrt{\ln(2)}} \end{cases} \qquad (33)$$

Figure 6 presents the evolution of the estimation functions in the case of a constant stiffness and constant damping. One can note the convergence to the nominal values $b_M$=1500N/m/s and $k_M$=15000N/m. Figure 7 presents the evolution of the estimation error. This error converges well towards zero thus confirming the theoretical results previously obtained. One also presents on this figure the road profile used for simulation.

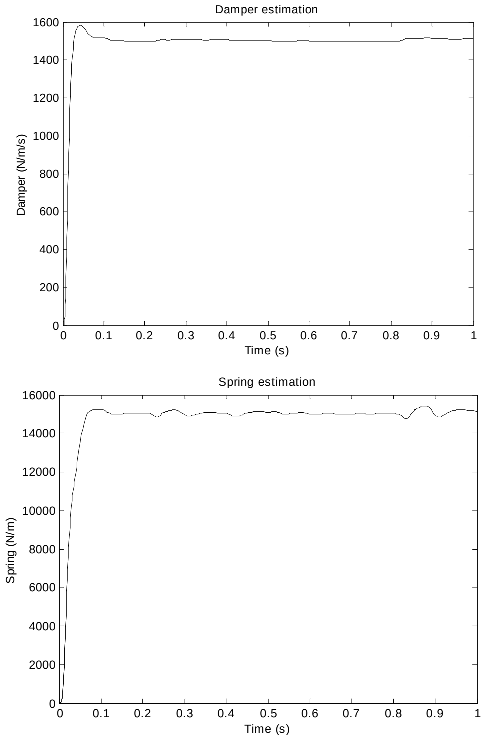



Fig. 6. *Estimation of the damper and spring.*

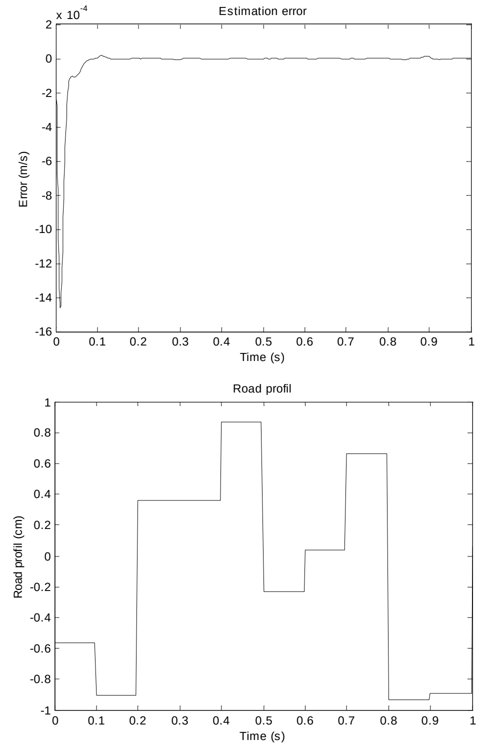



Fig. 7. *Estimation error and road profile.*

Figure 8 presents the tracking performance of the controlled system. The reference model is characterized by a stiffness of 10000N/m and a damping of 1000N/m/s. The controlled system behaves as a suspension whose characteristics are those of the reference model.

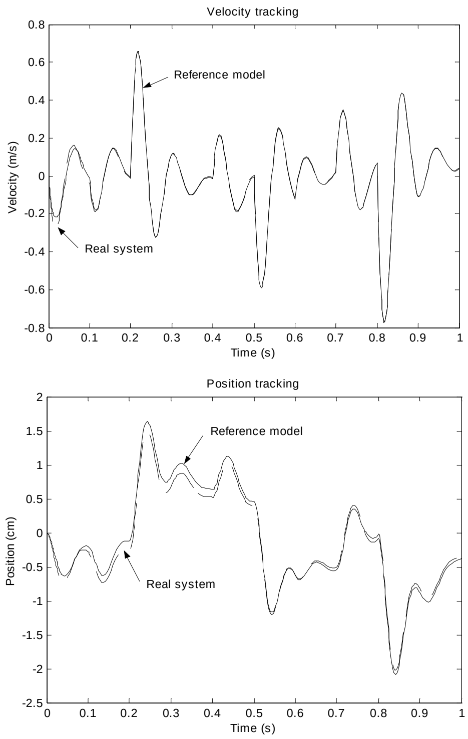



Fig. 8. *Tracking of the reference model.*

Figure 9 presents the evolution of the estimation functions in the case of a time varying stiffness and damping following the relations :

$$\begin{cases} k_M(t) = 15000 + 5000\sin(3t) \\ b_M(t) = 1500 + 500\sin(4t) \end{cases}$$

One can note the convergence towards the expressions of $b_M(t)$ and $k_M(t)$. Figure 10 presents the evolution of the estimation error, which converges well towards zero. This figure presents also the control applied to the system.

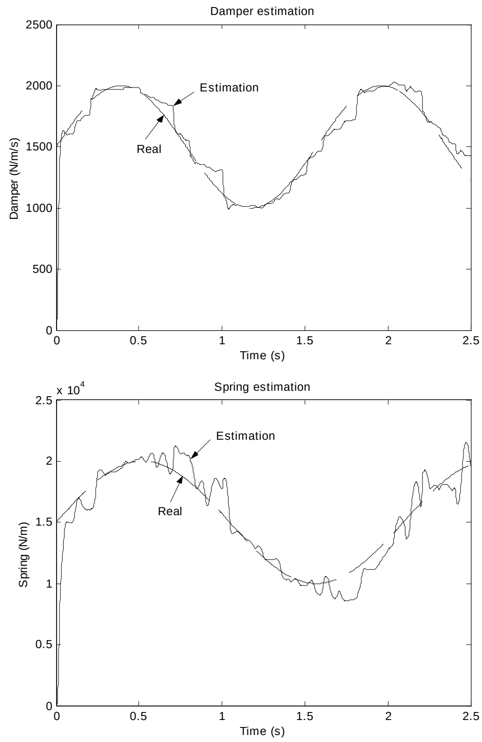



Figure 9 *Estimation of the damper and spring.*

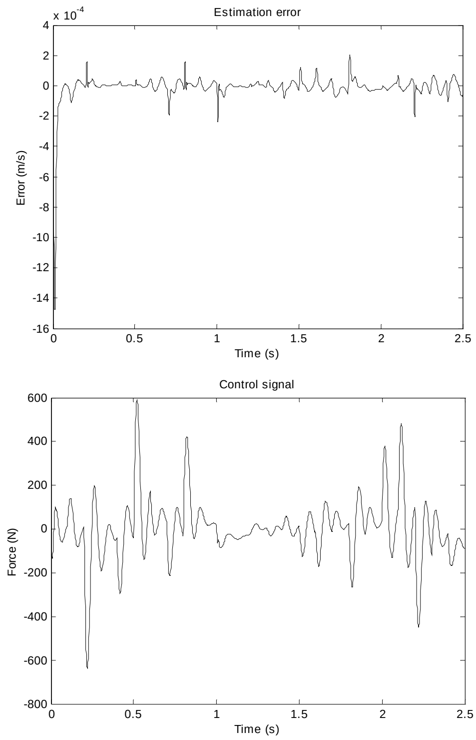



Fig. 10. *Estimation error and command applied to the system.*

Figure 11 presents the tracking performance of the controlled system. The reference model is, like previously, characterized by a stiffness of 10000N/m and a damping of 1000N/m/s. The controlled system behaves as a suspension whose characteristics are those of the reference model and independent of the parametric variation of the suspension.

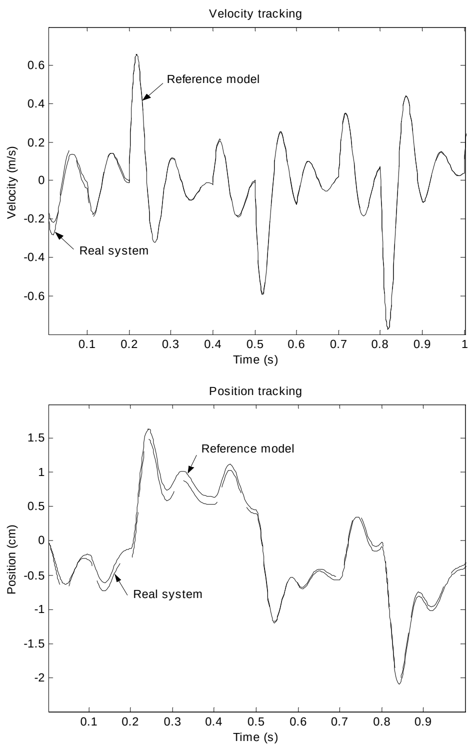



Fig. 11. *Tracking of the reference model.*

## 6. CONCLUSION

In this paper we have presented the modelling and effort control of an electromagnetic actuator intended for the control of a conventional suspension. The principle of the control rests on the compensation of the non linearity so as to obtain a suspension whose stiffness and damping are programmable. This type of control is very useful to adapt the suspension to the profile of the road and the different ways of driving. It is indeed difficult to design an optimal suspension in any circumstance. Such an approach allows us to obtain the best possible arrangement between the riding confort and the road-holding qualities, which is adjusted to meet the requirements of different types of driving and road profiles. It also compensates any other modifications such as those brought by the wear and tear of components

The main difficulty of this approach is the ignorance of the static characteristics of the components of the suspension. This difficulty is solved by using a continuous identification of the nonlinear functions characterizing the components of the suspension. This was made possible by the adoption of nonlinear parametrable functions having the property of universal approximator. The choice of fuzzy inference systems is well adapted thank to their capacity for approximation of the functions and their abilities to easily integrate qualitative knowledge. According to this principle the control developed in this work leads to a continuous nonlinear adaptive control.

The nonlinear adaptive control, which was suggested in this work, was applied successfully to a conventional suspension with the aim to obtain the stiffness and damping required by the user. The behavior of the closed loop system is very satisfactory, as much in the case of a suspension whose characteristics are static, which is not very realistic, as in the case or they evolve because of a change of operating conditions, which corresponds more to reality.

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