Quality Requirements Allocation Method Based on Industrial Data

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Abstract. Today, to cope with the complexity of the global organization, the industrial company needs to be more structured. New processes have to be developed due to more and more ambitious quality requirements. A new problematic arises: what is needed to offer to all customers a product that meets the quality requirements of a local market?

The main objective of this paper is to propose a quality requirements allocation method that matches the market specifications and the customer satisfaction. This is in contrast with the traditional allocation methods which are often time-consuming to implement or do not focus on the customer satisfaction for the definition of the quality targets. The proposed method is inspired from reliability allocation method and is formulated as a feasibility problem. In this context the notion of optimality of the solution is not being sought, the objective is "only" to find out a solution that satisfies the global target quality. This allows determining some local quality targets in accordance with industrial data.

Keywords: Quality Allocation, Fault Frequency, Product Life Cycle, Field Experience, Automotive Vehicle.

1 Introduction

1.1 Motivation and Objectives

This study deals with how to allocate a quality target to each part of the system considering the system complexity. This complexity comes from some technology constraints, the importance and the component function, the industrialization, the management of subcontractor, etc. The purpose of this research is to introduce a structured quality requirement allocation method that determines a quality objective for each component of a system, subject to the satisfaction of a global quality objective of the considered system.

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The method has to be suitable for industrial applications, that is to say:

- It has to be implemented easily,

- It has to be representative of reality,

- It has to take into account all the industrial criteria, in particular those that are critical for a company like cost, branding, etc.

Although an optimization approach could be used to solve this kind of problems, this seems not well suited in the considered context, indeed:

- The definition of an objective function and the constraints are often very time consuming due to system complexity. In addition, the formulation of the optimization problem is closely related to the considered system. In other words, the optimization problem formulated for a given system is not easily transposable to another one, and this is not very well accepted in an industrial context.

- The resulting optimization problem is non-convex, and so only a local optimum is guaranteed to be found. If in addition the problem is non-smooth then it may becomes intractable via standard optimization methods.

- Qualitative criterion cannot be easily taken into account via numerical optimization methods.

Due to these various limitations, an empirical approach is proposed. The purpose is not to obtain the optimal solution but a feasible one, *i.e.*, a solution that meets industrial needs.

1.2 Mathematical Formulation

Consider a system of n subsystems, each constituted by N_j independent components. By analogy to the reliability requirements allocation, the mathematical formulation of the quality requirements allocation at time instant t can be written as the following feasibility problem:

$$\begin{array}{lll} \text{find} & Q_j^s,\ Q_{ij}^c, & i=1,\cdots,N_j,\ j=1,\cdots,n \\ \text{subject to} & f_s(Q_1^s, Q_2^s \cdots, Q_n^s, t) \geq Q_S & \\ & f_j^s(Q_{1j}^c, Q_{2j}^c \cdots, Q_{N_j j}^c, t) \geq Q_j^s, & j=1,\cdots,n \end{array} \tag{1}$$

where $f_s(Q_1^s, Q_2^s \cdots, Q_n^s, t)$ represents the actual system quality, Q_j^s is the quality requirement allocated to the j^{th} subsystem, Q_S is the system quality objective, and f_s is the quality function allowing to compute the system quality knowing the quality of its constituents. Similarly, $f_j^s(Q_{1j}^c, Q_{2j}^c \cdots, Q_{N_j j}^c, t)$ represents the actual quality of the j^{th} subsystem, Q_{ij}^c is the quality requirement allocated to the i^{th} component of the j^{th} subsystem, and f_j^s is the quality function allowing to compute the subsystem quality knowing the quality of its components. The problem that has to be solved is how to choose the Q_j^s, Q_{ij}^c ,$i=1,\cdots,N_j$,

$j = 1, \cdots, n$, so that the system quality satisfy the objective Q_S. Thus formulated this problem has an infinite number of solutions. However, not all of them are interesting for practical applications and thus we have to select those that satisfy some additional constraints related to cost, branding etc. In addition, it is necessary to give a measure of the quality which represents the customer satisfaction. The approach proposed in this paper is to evaluate the customer satisfaction through the concept of fault frequency which represents the number of customer claims over a given period of time. These aspects will be considered further in the paper.

1.3 Related Work

The problem of reliability allocation has been addressed since the sixties, at that time allocation methods have been developed for military needs.

The AGREE method was created by the "Advisory Group of Reliability of Electronic Equipment" in 1957. It determines a minimum acceptable service life for each subsystem to meet the overall objective of the system [11]. The complexity and importance of each subsystem are considered.

Several approaches are detailed in the MIL-HDBK-338B (U.S. Department of Defense) in 1988. The Equal apportionment assigns the same reliability requirement to each component [9]. For the ARINC apportionment the allocated failure rate is proportional to the current failure rate. The Feasibility Of Objectives (FOO) technique allocates reliability requirements depending on four factors: the complexity, the state of art, the operating time during mission and the environment. All these methods assume the same hypothesis, the failure rate follows an exponential distribution and they are applied for series system.

Reliability allocation methods have been extensively used in industrial applications. In 1999, Kuo proposed a general model called Average weight for commercial applications [6]. In 2001, Wang introduced a reliability allocation method for computerized numerical control lathes design and development in automotive industry [13]. Seven criteria are considered: failure frequency, failure criticality, maintainability, complexity, manufacturing technology, working conditions and cost.

In 2009, Chang proposed the ME-OWA method based on the concept of OWA operators (Ordered weighted averaging) under maximal entropy to solve the problem described by the method FOO [2]. This method provides a conditional parameter α, ranging from 0.5 to 1 to take into account the decision maker optimism. Two years later, this method is improved by combining the decision-making laboratory test, the evaluation method (DEMATEL) and the ARINC method [7].

Tools for solving optimization problem have been applied for the reliability allocation complex applications since the nineteen's. Yang used a Genetic Algorithm to solve the reliability allocation applied to power plants in 1999 [16]. Another example is the reliability allocation to a multi-source network of stochastic flows. It determines an optimal policy for allocating resources that satisfies the demands at nodes while maximizing reliability [4]. Yalaoui introduced a model for minimizing resource consumption subject to a reliability constraint of a series-parallel system in 2005 [14]. In 2006, Zhang proposed a method called

Collaborative Allocation to solve the requirements allocation problem as the reliability and weight requirements when designing an aircraft [18]. The survey paper [6] classifies the optimal reliability allocation methods according to different criteria: the system structure, the problem formulation and the optimization techniques. Lisnianski introduced a method to evaluate the reliability of a multi-states system based on block diagrams in 2007 [8]. The paper of Ouzineb in 2008 described a simultaneous problem of allocation and redundancy for multi-states series-parallel systems solved by Tabu Search [10]. The paper of Tian presents an optimization problem of reliability and redundancy for a multi-state series-parallel system in 2009 [12]. The objective is to minimize the cost under the constraint of system availability. In 2009, Bicking proposed a reliability allocation optimization by Genetic Algorithm applied to the conception of a Security Instrumented System [1]. Yeh solved a redundancy allocation problem with an artificial bee colony algorithm in 2011 [17].

However, as seen before, the optimization approaches are problem dependent, and it is difficult to transpose the formulation obtained for a particular system to another one. Moreover, the formulation of an optimization problem is often a difficult task that requires the assistance of various experts of the system considered. Under these conditions optimization approaches seems not very easy to apply in an industrial context.

On the other hand, the traditional methods like the Equal Apportionment, AGREE, ARINC, FOO technique are quite flexible but not enough complete to answer the specifications of the problem. In particular, these approaches don't highlight the customer aspect. This is in contrast with the proposed method where this aspect is considered via the concept of Fault Frequency (see section 2.3). Overall, this paper introduces a new quality requirements allocation method that focuses on the customer aspect and includes also the warranty point of view.

The paper is organized as follows. Section 2 presents the main tools used in the proposed method. Section 3 gives a detailed description of the proposed quality requirements allocation method. In section 4, the proposed approach is applied on an automotive application example. Finally, section 5 concludes this paper.

2 Tools used for the proposed method

The proposed method is based on some basic tools which are now briefly reviewed.

2.1 Average Weight Method

The average weight method, proposed by Kuo is a flexible allocation model (see also [2]). An investigation is conducted to determine the most influential factors on system quality, such as complexity, environment, safety, or maintenance. Each factor X_{lij} is estimated on a scale ranging from 1 to 10, by expert judgement. According to this method, the j^{th}

average factor rate for the i^{th} subsystem, is defined as

$$Y_{ij} = \frac{1}{L} \sum_{l=1}^{L} X_{lij} \tag{2}$$

where X_{lij} is the j^{th} factor rate for the i^{th} subsystem done given by the l^{th} expert, and L is the number of experts. The weight w_i associated to the i^{th} subsystem is then defined as

$$w_i = \frac{\sum_{j=1}^{n} Y_{ij}}{\sum_{i=1}^{m} \sum_{j=1}^{n} Y_{ij}} \tag{3}$$

where n is the total number of subsystems and m is the total number of factors. Assuming a series system, the reliability objective R_s is distributed among all subsystems according to the weight w_i associated to each subsystem. So, the reliability requirement R_i, allocated to the i^{th} subsystem is given by

$$R_i = 1 - w_i(1 - R_S) \tag{4}$$

On the other hand, the failure rate λ_i, of the i^{th} subsystem is given by

$$\lambda_i = w_i \lambda_s \tag{5}$$

where λ_s is the system failure rate. As we will see, the method proposed in this paper is partially inspired by this model because of its simplicity and flexibility. However, as already said, one of the main contributions of the proposed approach is to include the customer satisfaction as well as some other after-sales data.

2.2 Product life cycle

The Product Life Cycle is represented in figure 1. For each life cycle phase, a factor that represents the quality impact on the product is considered. The State of Art Design factor represents the impact of the project maturity and the design complexity on the product quality. The Manufacturing and Assembly factor evaluates the impact of these processes on the failure probability. The Environment factor includes all the factors that can influence the component quality after the product sale such as the components interfaces, the specific vehicle application and working conditions (road, climate, etc.).

2.3 Fault frequency

In the proposed method, the quality of a given component is evaluated by the concept of fault frequency F, which represents the number of customer claim per product N_{claim}, over a given period of time T. The quantity F is then defined as follows

$$F = \frac{N_{claim}}{T} \tag{6}$$

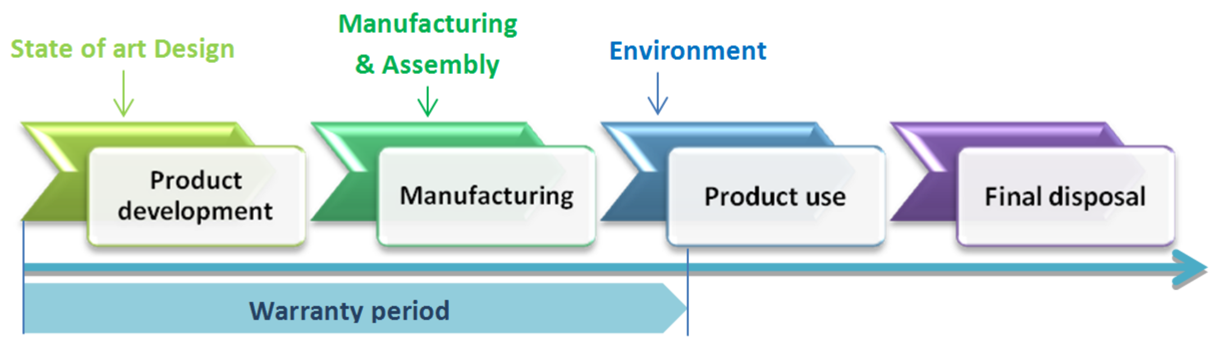

Figure 1: Factors which impact the product quality during each phase of the product life cycle.

This industrial quality measurement is chosen to highlight the customer aspect. Indeed, it measures the customer claim. It does not only consider the failure of a product but also the customer satisfaction. If the product has no failure and the customer is not satisfied of that product then he can make a claim that increases the Fault Frequency.

Field experience feedbacks are considered thanks to the warranty cost and the claim occurrence. To consider the system cost, it is not possible to sum up all the components average claim costs because it overestimates the repair cost. Indeed, it is sometimes cheaper to replace the overall sub-system than the component. This is why another criterion is used to represent the impact of the warranty costs to the sub-system level. The warranty cost can be expressed quantitatively as being the average claim cost in euros for components or qualitatively by considering the branding, the criticality and the cost for sub-systems.

2.4 DEMATEL Methodology

It is well known that the more a component is subjected to the influence of the other components, the more the risk of failure is important. A criterion inspired from the DEMATEL (Decision Making Trial and Evaluation Laboratory) matrix considers the system interactions [7]. The DEMATEL method converts the relationship between the causes and effects of criteria into an intelligible structural model. It takes into account the direct and indirect influences among multiple factors. For the proposed allocation method, the DEMATEL method is used only to consider the influences between systems or components.

According to the DEMATEL method, we built an influences matrix, denoted Z. The entry z_{ij} of this matrix, represents the degree with which the criterion $crit_i$ affects the criterion $crit_j$. The matrix Z has the following structure

$$\begin{bmatrix} 0 & z_{12} & \cdots & z_{1n} \\ z_{12} & 0 & \cdots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & 0 \end{bmatrix} \tag{7}$$

The entries of the Z matrix can have four values that represent the influence level

- 0 is chosen for no influence,
- 1 for low influence,
- 2 for medium influence,
- 3 for high influence.

Let D_i be the sum of the elements of the i^{th} row of Z, and C_j the sum of the j^{th} column of the matrix Z, we have

$$D_i = \sum_{j=1}^{n} z_{ij}, \quad C_j = \sum_{i=1}^{n} z_{ij} \tag{8}$$

The quantity D_i represents the sum of the direct influences given by the component or the subsystem. Similarly, C_j represents the sum of the direct influences received by the component or the subsystem.

3 Proposed Method

3.1 quality Requirement Allocation Model

The proposed quality allocation method procedure is organized into several steps and is described as follows

- **Step 1.** Build a functional breakdown of the studied system into subsystems. Then, build a functional breakdown of the subsystems into components (see figure 3). It is also possible to have a more complicated breakdown into four or five levels.

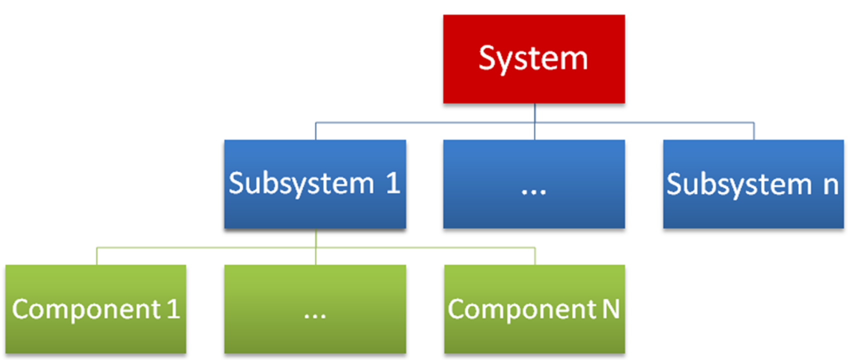

Figure 2: Functional breakdown of the system into subsystems and functional breakdown of each subsystem into components.

- **Step 2.** The quality objective of the complete system F_S is defined by Strategy, Marketing and Quality.

- **Step 3.** Evaluate the criteria for each subsystem to find the weight of the j^{th} subsystem W_j^c and each component to find the weight of the i^{th} component associated to the j^{th} subsystem W_{ij}^s. As we will see, these weights are obtained by using the following data

  - Fault Frequency (quantitative),
  - Warranty costs (quantitative),
  - Life cycle factor(qualitative):
    - State of art Design,
    - Manufacturing,
    - Environment.

  The method used to obtain W_{ij}^c and W_j^s is presented in paragraphs 3.1.1, 3.1.2 and 3.1.3.

- **Step 4.** Compute the j^{th} allocated Fault Frequency $[F_j^s]_{target}$ for the subsystem level and then for the component level (see figure 3). The Fault Frequency allocated to the j^{th} subsystem is given by

$$[F_j^s]_{target} = W_j^s F_S \tag{9}$$

  where W_j^s is the weight associated to the the j^{th} subsystem, and F_S is the system Fault Frequency objective. Similarly, the Fault Frequency allocated to the i^{th} component of the j^{th} subsystem, is given by

$$[F_{ij}^c]_{target} = W_{ij}^c [F_j^s]_{target} \tag{10}$$

  where W_{ij}^c is the weight associated to the the i^{th} component of the considered j^{th} subsystem.

In the following paragraphs, the method used to obtain W_{ij}^c and W_j^s is described. The paragraph 3.1.1 explains how to obtain the product life cycle factor family. The paragraph 3.1.2 presents the process to compute the field experience factor family and 3.1.3 is dedicated to the computation of W_{ij}^c and W_j^s. The figure 4 summarizes the way to obtain the final weight.

3.1.1 Product Life Cycle Factor Family

The State of art Design S, the Manufacturing and Assembly M and the Environment E factors are three criteria that influence quality during the Product Cycle Life. Each criterion is qualitatively evaluated by experts between 1 and 10. The value 10 is assigned when

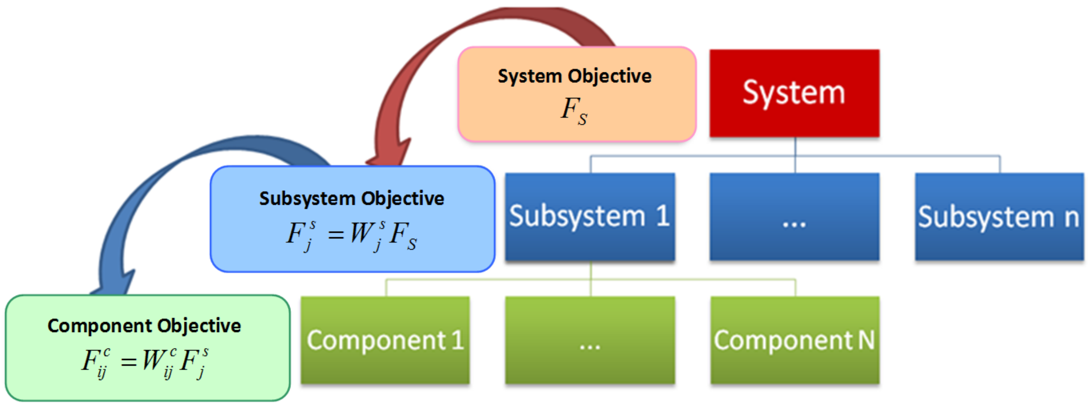

Figure 3: Allocation principle: The system Fault Frequency objective is allocated to subsystems and each subsystem Fault Frequency objective is allocated to components.

the failure risk is the highest (non-mature development, non-controlled process, severe environment). The value 1 is assigned for the lowest influence of criterion on the failure risk (mature development, controlled process, non-severe environment). These factors are combined to define the so called Life Cycle Factors (LCF). The LCF is defined at the component level as well as at the sub-system level.

Component Level. For the component level, the Life Cycle Factor of the i^{th} component associated with the j^{th} subsystem, denoted L_{ij}^c, is defined as the following weighted average

$$L_{ij}^c = \frac{aS_{ij} + bM_{ij} + cE_{ij}}{a + b + c} \tag{11}$$

where S_{ij} is the State of art Design factor, M_{ij} is the Manufacturing and Assembly factor, E_{ij} is the Environment factor, i is the component index, j represents the subsystem index, a is the Design factor weight, b is the Manufacturing factor weight, and c is the Environment factor weight. These weights are useful when the decision maker wants to give more importance for one factor compared to the others. In our case, we have chosen to assign 1 to a, b, c, because we have considered that the three factors have the same importance. Alternatively, these weights can be determined by the preference order approach presented in Appendix A1. The life cycle factor is then normalized as follows

$$\bar{L}_{ij}^c = \frac{L_{ij}^c}{\sum_{i=1}^{N_j} L_{ij}^c} \tag{12}$$

where N_j is the number of components of the considered j^{th} subsystem.

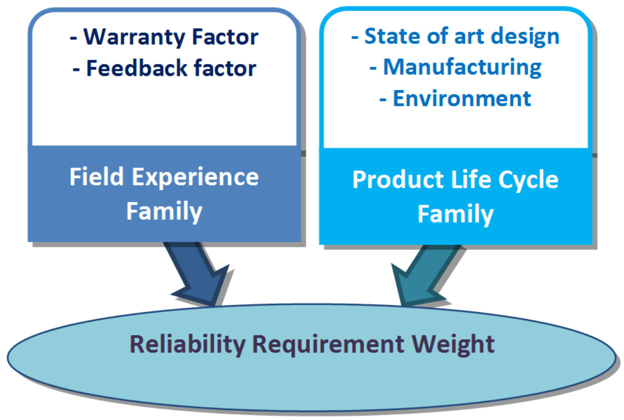

Figure 4: Definition of the quality Requirements weight with Field Experience criteria family and Product Life Cycle criteria Family.

Subsystem Level. For the subsystem level, the Life Cycle Factor of the j^{th} subsystem, denoted λ_j^s, is defined as follows

$$\lambda_j^s = \sum_{i=1}^{N_j} L_{ij}^c \tag{13}$$

where L_{ij}^c is the Life Cycle Factor of the i^{th} component attached to the j^{th} subsystem. For the subsystem level an interaction factor is added to the Life Cycle factor. The interaction factor is the normalization of the influence matrix columns sum (see section 2.4 DEMATEL methodology):

$$I_j = \frac{\sum_{i=1}^{n} z_{ij}}{\sum_{j=1}^{n}\sum_{j=1}^{n} z_{ij}} \tag{14}$$

where I_j is the Interaction Factor of the j^{th} subsystem, z_{ij} is the weight of influence of the i^{th} subsystem on the j^{th} subsystem, and n is the number of subsystems.

The Life Cycle Factor of the j^{th} subsystem that takes into account the interaction factor is then defined as

$$L_j^s = d\lambda_j^s + eI_j \tag{15}$$

where d is the weight of the life cycle factor of the j^{th} subsystem, and e is the weight of the impact factor factor of the j^{th} subsystem. These weights can be used by the decision maker to give more importance for one factor compared to the other. Finally, the life cycle factor is normalized as follows

$$\bar{L}_j^s = \frac{L_j^s}{\sum_{j=1}^{n} L_j^s} \tag{16}$$

where n is the number of subsystems of the considered system.

3.1.2 Field Experience Factor Family

In this section we introduce successively the Warranty factor, the Fault Frequency factor and the Field experience factor.

■ Warranty Factor

As for the life cycle factor case, the Warranty factor is defined at the component level as well as at the subsystem level.

Component Level. The warranty cost is defined as the sum of the repaired cost, the spare cost and the additional costs for each component (see (17)). All these data are available from the industrial feedbacks. All these costs are available from the industrial data.

$$\Psi^c_{ij} = R_{ij} + S_{ij} + A_{ij} \tag{17}$$

where Ψ^c_{ij} is the Warranty cost of the i^{th} component associated to the j^{th} subsystem, R_{ij} is the Repaired cost of the i^{th} component, S_{ij} is the spare cost of the i^{th} component, and A_{ij} is the additional cost of the i^{th} component.

The Warranty cost is then scaled according to a given cost scale. In fact, this transformation is used to represent a given cost interval by a score reflecting the goodness of the warranty cost (a lower cost receives a better score). This allows us to take into account uncertainties about the warranty costs. In addition, this transformation allows to have a warranty cost score independent of the industrial application domain (automotive, aircraft, nuclear power plants etc.). The only thing that changes between the various industrial application domains is the mapping between the cost intervals and the scores. This mapping is done according to the industrial practice existing in a given application domain. Table 1 is given as an example of such a transformation and will be used in the automotive application provided later on.

Table 1: Example of Warranty Cost scale which summarizes the industrial practice.

Score	10	9	8	7	6	5	4	3	2	1
Interval Cost (Euros)	< 100	100 to 199	200 to 249	250 to 399	400 to 499	500 to 699	700 to 999	1000 to 1499	1500 to 2999	≥ 3000

After scaling, the warranty factor is normalized as follows

$$\bar{\Psi}^c_{ij} = \frac{\widetilde{\Psi}^c_{ij}}{\sum_{i=1}^{N_j} \widetilde{\Psi}^c_{ij}} \tag{18}$$

where $\widetilde{\Psi}^c_{ij}$ represents the score attributed to Ψ^c_{ij}.

Subsystem Level. A qualitative factor is estimated based on the after sales expertise. The cost, the branding and the criticality are considered. The warranty cost of the j^{th} subsystem is defined as

$$\Psi_j^s = fB_j + gC_j^r + hC_j^o \tag{19}$$

where B_j is the branding qualitative factor, C_j^r is the criticality qualitative factor, C_j^o is the cost qualitative factor, f is the Branding factor weight, g is the Criticality factor weight, and h is the Cost factor weight. These three criteria are qualitatively evaluated by after-sales experts ranging from 1 to 10. So the sum is between 3 and 30. The value 1 is assigned when the criterion is considered as very important by the customer, in other words when the customers want to have a high quality. The Branding factor measures the subsystem failure impact on Branding. The Criticality factor represents the importance of a failure on the subsystem operation. The Cost factor represents the warranty costs. For this factor, it is possible to build a scale like Table 1. The value 1 is then assigned when the system must be replaced whereas the value 10 is assigned for a cheaper repair.

The weights f, g, h, can be used to give more importance for one factor compared to others. In our case, we have chosen to assign 1 to f, g, h, because we have considered that the three factors have the same importance. Alternatively, these weights can be determined by the preference order approach presented in Appendix A1. Finally, the warranty factor is normalized as follows

$$\bar{\Psi}_j^s = \frac{\Psi_j^s}{\sum_{j=1}^{n} \Psi_j^s} \tag{20}$$

■ Fault Frequency Factor

The subsystem Fault Frequency is given by the sum of all the components Fault Frequency, *i.e.*

$$F_j^s = \sum_{i=1}^{N_j} F_{ij}^c \tag{21}$$

where F_{ij}^c is the Fault Frequency of the i^{th} component associated to the j^{th} subsystem, and F_j^s is the Fault Frequency of the j^{th} subsystem. As usual, the Fault Frequency factor is normalized as follows:

- Component Level

$$\bar{F}_{ij}^c = \frac{F_{ij}^c}{\sum_{i=1}^{N_j} F_{ij}^c} \tag{22}$$

- Subsystem Level

$$\bar{F}_j^s = \frac{F_j^s}{\sum_{j=1}^{n} F_j^s} \tag{23}$$

■ **Field Experience Factor**

The Field Experience factor is defined at the component level as well as at the subsystem level.

Component Level. The Field Experience factor of the i^{th} component associated to the j^{th} subsystem is defined as

$$\Phi_{ij}^{c} = \bar{\Psi}_{ij}^{c} \bar{F}_{ij}^{c} \tag{24}$$

where $\bar{\Psi}_{ij}^{c}$ is given by (18), and $\bar{F}_{ij}^{c}$ is given by (22). The Field experience factor is then normalized as follows

$$\bar{\Phi}_{ij}^{c} = \frac{\Phi_{ij}^{c}}{\sum_{i=1}^{N_j} \Phi_{ij}^{c}} \tag{25}$$

Subsystem Level. The Field Experience factor of the j^{th} subsystem is defined as

$$\Phi_{j}^{s} = \bar{\Psi}_{j}^{s} \bar{F}_{j}^{s} \tag{26}$$

where $\bar{\Psi}_{j}^{s}$ is given by (20), and $\bar{F}_{j}^{s}$ is given by (23). The Field experience factor is then normalized as follows

$$\bar{\Phi}_{j}^{s} = \frac{\Phi_{j}^{s}}{\sum_{j=1}^{n} \Phi_{j}^{s}} \tag{27}$$

3.1.3 Quality Requirement Weight

The quality requirement weights are evaluated at the component level as well as at the subsystem level.

Component Level. Let K_{ij}^{c} be an intermediate factor allowing to take into account the component Life Cycle factor $\bar{L}_{ij}^{c}$ see (12) and the Field experience factor $\bar{\Phi}_{ij}^{c}$ see (25), we have

$$K_{ij}^{c} = o\bar{L}_{ij}^{c} + p\bar{\Phi}_{ij}^{c} \tag{28}$$

where o is the weight of the Life Cycle factor, and p is the weight of the Field Experience factor. The weights o, p, can be used to give more importance for one factor compared to the other. Alternatively, these weights can be determined by the preference order approach presented in Appendix A1. The quality requirement weight of the i^{th} component associated to the j^{th} subsystem is then defined as

$$W_{ij}^{c} = \frac{K_{ij}^{c}}{\sum_{i=1}^{N_j} K_{ij}^{c}} \tag{29}$$

Subsystem Level. Let K_{i}^{s} be an intermediate factor allowing to take into account the subsystem Life Cycle factor $\bar{L}_{j}^{s}$ see (16) and the Field experience factor $\bar{\Phi}_{j}^{s}$ see (27), we have

$$K_{j}^{s} = o\bar{L}_{j}^{s} + p\bar{\Phi}_{j}^{s} \tag{30}$$

where o and p have the same meaning as before. The quality requirement weight of the j^{th} subsystem is then defined as

$$W_j^s = \frac{K_j^s}{\sum_{j=1}^{n} K_i^s} \tag{31}$$

3.2 Computation Synthesis

The proposed procedure is summarized in figure 5. The main steps are as follows.

1. First, the input data are recovered for subsystems and components.
2. Then, the weight of each item is computed.
3. The system quality objective is allocated into each subsystem depending on the subsystems weights. For each subsystem, the quality objective is allocated into each components of the subsystem depending on the component weights.

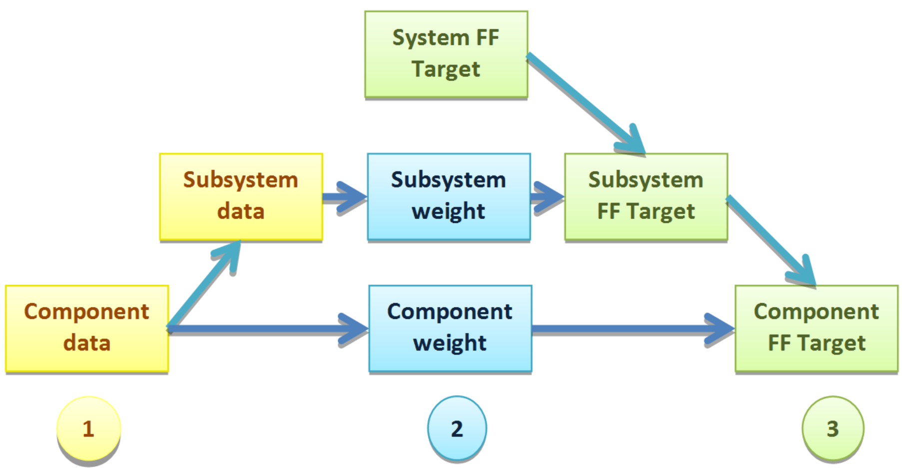

Figure 5: Computation procedure to allocate the system FF target to subsystems and then to the components.

4 Case Study

The proposed method is applied on a truck vehicle. The details of the engine subsystems and the Piston Cylinder Unit components are presented. Since the data are confidential, false input data are used for the application presented Table 2 and Table 3.

Table 2: Engine Subsystem inputs. The various weights have been taken as: $a = b = c = f = g = h = o = 1$, $e = 0.04$, and $p = 2$. The engine FF objective is 20.

Subsystems	L_j^s	I_j	F_j^s	B_j	C_j^r	C_j^o
Air & gas system	120	8	3	7	9	5
Fuel injection system	110	7	1	5	5	7
Combustion system	0	0	0	0	0	0
Power Cylinder Unit system	24	8	0.5	7	7	7
Crank system	90	6	1	5	9	5
Valve train system	30	4	0.5	4	9	6
Crank Case ventilation system	15	4	1	3	8	3
Auxiliaries drive system	60	3	2	3	7	9
Starter system	5	1	0.3	7	7	5
Engine Lubrication system	75	7	6	6	7	3

Table 3: Components Inputs for Power Cylinder Unit System. The various weights have been taken as: $a = b = c = f = g = h = o = 1$, and $p = 2$

Power Cylinder Unit system ($j = 4$)	S_{ij}	M_{ij}	E_{ij}	Ψ_{ij}^c	F_{ij}^c
Cylinder Liners	1	4	9	11000	0,06 %
Sealing rings	3	5	2	1500	0,03 %
Piston	2	3	10	9000	0,05 %
Piston rings	5	7	8	5000	0,01 %
Piston pin	4	2	7	2000	0,01 %

In this application we want to give more importance for the Field experience factor than to the product life cycle factor, that is to say, the experience and the cost are highlighted. The results are presented Table 4 and Table 5.

The obtained quality requirements have been compared to those obtained via the after sales service. It has been observed that the quality targets are close to those observed by the after sales service. To give an idea, the RMS value error calculated for the systems Fault Frequency and the components Fault Frequency, have been found to be $E_s = 0.85$ and $E_j^c = 0.16$ respectively. The quantity E_s and E_j^c are defined as follows

$$E_s = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left([F_j^s]_{target} - [F_j^s]_{obs}\right)^2}, \quad E_j^c = \sqrt{\frac{1}{N_j}\sum_{i=1}^{N_j}\left([F_{ij}^c]_{target} - [F_{ij}^c]_{obs}\right)^2} \tag{32}$$

where $[F_j^s]_{obs}$ is the actual fault frequency, in %, observed on j^{th} subsystem over twelve months. Similarly, $[F_{ij}^c]_{obs}$ is the actual fault frequency, in %, observed on i^{th} component associated to the j^{th} subsystem, over twelve months. In addition, the allocated Fault Frequency have been found to be systematically bigger than those actually observed via the after sales service. This mean that the proposed approach is slightly pessimistic, and this is an interesting property for the design of industrial products.

Table 4: Allocation of the Engine combustion system quality target to subsystems

Engine Combustion system	$[F_j^s]_{target}$	Subsystem weight W_j^s
Air & gas system ($j = 1$)	3.43%	17.2%
Fuel injection system ($j = 2$)	2.30%	11.5%
Combustion system ($j = 3$)	0.00%	0.0%
Power Cylinder Unit system ($j = 4$)	0.64%	3.2%
Crank system ($j = 5$)	1.91%	9.6%
Valve train system ($j = 6$)	0.78%	3.9%
Crank Case ventilation system ($j = 7$)	1.35%	6.8%
Auxiliaries drive system ($j = 8$)	2.32%	11.6%
Starter system ($j = 9$)	0.30%	1.5%
Engine Lubrication system ($j = 10$)	6.96%	34.8%

Table 5: Allocation of the Power Cylinder Unit quality target to components.

Power Cylinder Unit system ($j = 4$)	$[F_{ij}^c]_{target}$	Component weight W_{ij}^c
Cylinder Liners ($i = 1$)	0.18%	28.1%
Sealing rings ($i = 2$)	0.17%	26.6%
Piston ($i = 3$)	0.16%	25.0%
Piston rings ($i = 4$)	0.08%	12,5%
Piston pin ($i = 5$)	0.05%	7.8%

5 Conclusion

For four decades, numerous studies have been published about the reliability allocation; this interest reflects the strategic importance and the complexity of reliability allocation problem for many applications. Two types of allocation methods are developed in the literature: The traditional allocation methods like the AGREE apportionment or ARINC apportionment that are too simple to answer the specifications of the problem and the optimization techniques applied to the reliability allocation problem, like meta-heuristics techniques, which are too time consuming for industrial applications. The FOO technique and the average weight methods provide a partial answer to our problem.

The proposed method allows to introduce several improvements: First, the model highlights the customer aspect by the chosen quality measurement unit, the Fault Frequency, and by the warranty factors. Secondly, the criteria definition can be different depending on the allocation level considered. Indeed, a criterion can be relevant for a specific level and not for an upper level. The best solution depends on the problem scale. Thirdly, two main factor families are considered to define the weight of each system part for the quality target allocation. The product life cycle family considers the criteria that will impact the quality during each life cycle phase and the field experience family uses the company experience and the warranty cost. Then, the interactions between subsystems are taken into account thanks to a criterion inspired by the DEMATEL methodology. Finally the weight of each factor makes the method more flexible. The new method has been successfully applied on an automotive example. So, the purpose of this work is achieved: a quality requirements allocation method is proposed and a quality objective for each component is determined.

The proposed method can be extended to the case of a non-constant failure rate. This case can be interesting to be studied in particular for long observation time. The hypothesis about the constant failure rate seems to be not relevant in this case. Another line of research would be to change the way of measuring quality, *i.e.*, to consider alternative approach to the Fault Frequency measure.

Appendix

A1. Preference Order Weights Selection

In the proposed method, we have to associate a weight to some given factors in accordance with the importance placed in these factors. These weights can be fixed through experts advice, but this is not always possible. In such situation it is better to choose among all possible weight distributions the one that is less arbitrary. This last objective can be reached via the maximum entropy principle (see for instance [5] for a good coverage of the maximum entropy principle).

According to the maximum entropy principle, the less arbitrary weight distribution is the one that satisfy the constraints (in our case the preference order) while maximizing the entropy.

More formally, consider n factors $f_1, f_2, \cdots, f_n$, arranged in the preference order from the most important to the less important. For instance the arrangement (f_1, f_2, f_3) means that a more importance is accorded to the factor f_1 with respect to f_2 and f_3, and a more importance is accorded to f_2 with respect to f_3. Now, we want to define a weighted average index, denoted I_F, that reflects the preference order of the factors. This index is written as

$$I_F = \frac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} w_i} \tag{33}$$

where w_i is the weight associated to the factor f_i. Since f_i is considered as more important than f_{i+1} we must satisfy the following constraint

$$w_i > w_{i+1}, \quad i = 1, \cdots, n-1 \tag{34}$$

The problem is then to determine the less arbitrary weight distribution $w_1, w_2, \cdots, w_n$ satisfying the constraint (34). Note that the constraints (34) can be satisfied with w_i slightly bigger than w_{i+1}. In order to impose a stronger dominance it is better to satisfy constraints of the form

$$w_i \geq \alpha_i + w_{i+1}, \quad i = 1, \cdots, n-1 \tag{35}$$

with α_i a sufficiently large positive number satisfying

$$\alpha_i \geq \beta\alpha_{i+1}, \quad i = 1, \cdots, n-1 \tag{36}$$

where $\beta > 1$ is a user defined coefficient. The weighted average index I_F defined in (33) can be equivalently rewritten as

$$I_F = \sum_{i=1}^{n} \tilde{w}_i f_i, \quad \tilde{w}_i = \frac{w_i}{\sum_{i=1}^{n} w_i} \tag{37}$$

Note that $\sum_{i=1}^{n} \tilde{w}_i = 1$. So, I_F is a convex combination of the factor f_i. According to what has been said before, we have to find the less arbitrary weight distribution $\tilde{w}_1, \tilde{w}_2, \cdots, \tilde{w}_n$, satisfying the constraints

$$\begin{cases} \tilde{w}_i \geq \alpha_i + \tilde{w}_{i+1}, & i = 1, \cdots, n-1 \\ \alpha_i \geq \beta\alpha_{i+1}, & i = 1, \cdots, n-1 \\ \sum_{i=1}^{n} \tilde{w}_i = 1 & \end{cases} \tag{38}$$

By using the maximum entropy principle both for the $\tilde{x}_i$'s and the α_i's, we have to solve the following optimization problem

$$\begin{array}{ll} \text{maximize} & -\left(\sum_{i=1}^{n} \tilde{w}_i \log(\tilde{w}_i) + \sum_{i=1}^{n-1} \alpha_i \log(\alpha_i)\right) \\ \text{subject to} & \alpha_i \geq \beta\alpha_{i+1}, \quad i = 1, \cdots, n-1 \\ & \tilde{w}_i \geq \alpha_i + \tilde{w}_{i+1}, \quad i = 1, \cdots, n-1 \\ & \sum_{i=1}^{n} \tilde{w}_i = 1 \end{array} \tag{39}$$

Note that this formulation is a convex programming problem, this is in contrast to the OWA operator introduced in [15]. So, the problem (39) can be efficiently solved (the global optimum is guaranteed to be found in polynomial time) via existing convex solvers such as for instance the cvx package [3].

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