A simple robust PI/PID controller design via numerical optimization approach

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Abstract. This paper presents a simple but effective method for designing robust PI or PID controller. The robust PI/PID controller design problem is solved by the maximization, on a finite interval, of the shortest distance from the Nyquist curve of the open loop transfer function to the critical point -1. Simulation studies are used to demonstrate the effectiveness of the proposed method.

Keywords: PID controller; Robustness; Numerical optimization; Time-delay.

1 Introduction

The proportional-integral (PI) and proportional-integral-derivative (PID) controllers are widely used in many industrial control systems for several decades since Ziegler and Nichols proposed their first PID tuning method. This is because the PID controller structure is simple and its principle is easier to understand than most other advanced controllers. On the other hand, the general performance of PID controller is satisfactory in many applications. For these reasons , the majority of the controllers used in industry are of PI/PID type.

Most of real plant operate in a wide range of operating conditions, the robustness is then an important feature of the closed loop system. When this is the case, the controller has to be able to stabilize the system for all operating conditions. To this end, it is possible to employ an internal-model-based PID tuning method (Morari and Zafirou, 1989; Chien and Fruehauf, 1990). However, this method gives very slow response to load disturbance for lag-dominant processes because of the pole-zero cancellations inherent in the design methodology (Astrom and Hagglünd, 1995). Another popular approach with similar emphasis is the tuning of PI or PID controller by the gain and phase margin specifications (Ho & al., 1995; Astrom and Hagglund, 1988). Gain margin and phase margin have always served as important measures of robustness. It is well known that phase margin is related to the damping of the system, and can therefore also serve as a performance measure (Franklin and al., 1986). In this way, numerous progress has been made to improve the performances of the PI/PID control (Hang & al., 2002). In particular, tuning methods based on optimization approach have recently received more attention in the literature, with the aim of ensuring good stability robustness of the controlled system (Hwang and Hsiao, 2002; Ge & al., 2002). However these new methods are not easy to use for the operating engineer who is the main user of the PI/PID controller.

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The objective of this paper is to propose a novel robust PI or PID controller which is simple and easy to use.

The paper is organized as follows. Section 2 presents the process models used for the synthesis of the PI or PID controller. In section 3, the robust PI/PID controller design problem is formulated and solved by the maximization, on a finite interval, of the shortest distance from the Nyquist curve of the open loop transfer function to the critical point -1. Simulation studies are conducted in section 4 and comparison with the works of other authors are given. Section 5 concludes this paper.

2 Process models

The industrial processes are of an extreme variety. Nevertheless, a very broad class is characterized by aperiodic response. This important category of industrial systems can be represented by a first order plus dead time model, as follows:

$$G(s) = \frac{ke^{-t_0 s}}{1+\tau s} \tag{1}$$

Note that the above process model is only used for the purpose of simplified analysis. The actual process may have multiple lags, non-minimum phase zero, etc. Another important class of industrial processes is characterized by non aperiodic response. This category of processes can be represented by a second-order plus dead-time model, as follows:

$$G(s) = \frac{ke^{-t_0 s}}{s^2 + a_1 s + a_0} \tag{2}$$

Many identification techniques can be used to obtain first-order plus dead-time or second-order plus dead-time model for PI/PID control (Despand and Ash, 1988; Astrom & al., 1993). A simple method is based on the analysis of the open-loop step response. The first-order plus dead-time model (1) is obtained as follows:

$$\begin{cases} k = y_\infty \\ t_0 = 2.8t_1 - 1.8t_2 \\ \tau = 5.5(t_2 - t_1) \end{cases} \tag{3}$$

where y_∞ is the final value of the step response of the process, t_1 (respectively t_2) is the time where the output attains 28% (respectively 40%) of its final value. For the second-order plus dead-time model (2), the parameters are obtained as follows:

$$\begin{cases} k = y_\infty \\ t_0 : \text{is the apparent time delay} \\ a_1 = \dfrac{2|\ln(D_1)|}{\pi t_p}, \quad a_0 = \dfrac{\pi^2 + \ln(D_1)^2}{\pi^2 t_p^2} \end{cases} \tag{4}$$

where D_1 is the first overshoot for the unit step response of the process and t_p is the corresponding time. Alternatively, these models can be derived from relay feedback method (Astrom and Hagglund, 1995, 1988). This method can be extended to open-loop unstable processes (Scali & al., 1999; Marchetti & al., 2001).

3 Robust PI/PID controller design

In this section, the robust PI/PID controller design problem is formulated and solved via numerical optimization method.

3.1 Problem statement

Consider the PID feedback control system shown in 1, in which $G(s)$ represents the transfer function of the process model (1) or (2) and $K(s)$ is the transfer function of the standard PI/PID controller :

$$\begin{cases} \text{PI}: & K(s) = k_p + \dfrac{k_i}{s} \\ \text{PID}: & K(s) = k_p + \dfrac{k_i}{s} + k_d s \end{cases} \tag{5}$$

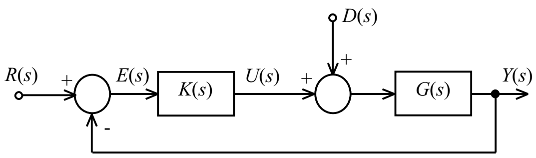

FIG. 1 – *Block diagram of the PID feedback control system*

For this control system, the sensitivity function $S(s)$ and complementary sensitivity function $T(s)$ which is the transfer function of the closed loop system, are respectively, defined by :

$$S(s) = \frac{1}{1 + K(s)G(s)} = \frac{1}{1 + L(s)} \tag{6}$$

where $L(s) = K(s)G(s)$ is the open-loop transfer function, and :

$$T(s) = 1 - S(s) = \frac{L(s)}{1 + L(s)} \tag{7}$$

The quantity $|T(j\omega)|$ represents the input-output gain at the frequency $2\pi/\omega$, for a PI/PID controller this gain is equal to one in the low frequency domain, that is the steady state error is equal to zero. The quantity $M_p = \max_\omega |T(j\omega)|$ is the peak magnitude of the frequency response of the closed-loop system. It is well known that M_p is related to the overshoot for the step response of the closed-loop system. In order to impose good transient response it is necessary to have :

$$M_p \leqslant M_p^+ \tag{8}$$

where $M_p^+ > 1$ is the upper bound of the maximum of the complementary sensitivity function. In an equivalent manner the following constraint is required :

$$D_1 \leqslant D_1^+ \tag{9}$$

where D_1 is the first overshoot of the step response and D_1^+ is the upper bound value of this overshoot. It is then possible to introduce a lower bound pseudo damping factor ζ_m, which is related to the upper bound of the first overshoot by the relation :

$$\zeta_m = \frac{|\ln(D_1^+)|}{\sqrt{\pi^2 + \ln(D_1^+)^2}} \tag{10}$$

the relation between M_p^+ and the lower bound pseudo damping factor ζ_m, is given by (Di Stefano & al., 1975):

$$M_p^+ = \frac{1}{2\zeta_m\sqrt{1-(\zeta_m)^2}} \tag{11}$$

For a good transient response it is then required that:

$$\zeta \geqslant \zeta_m \tag{12}$$

where ζ is the pseudo damping factor of the closed-loop system. The quantity $1/|S(j\omega)|$ represents the distance between the Nyquist curve of the open-loop transfer function $L(s)$ and the critical point -1 at the frequency $2\pi/\omega$. The minimum of this distance represents then a good measure of the stability margin.

Consider an additive error model of the open loop transfer function $\Delta L(s)$, the influence of this error on the closed-loop transfer function can be deduced from the first order Taylor series expansion :

$$T(L(s) + \Delta L(s)) = T(L(s)) + \frac{\partial T(s)}{\partial L(s)}\Delta L(s) \tag{13}$$

which gives the well known result :

$$\frac{\Delta T(s)}{T(s)} = S(s)\frac{\Delta L(s)}{L(s)} \tag{14}$$

The quantity $\max_\omega |S(j\omega)|$ represents then a good evaluation of the robustness in the face of model uncertainties. The sensitivity function $S(s)$ appears also in the transfer function of the input disturbance $D(s)$ to the output $Y(s)$:

$$Y(s) = G(s)S(s)D(s) \tag{15}$$

The quantity $\max_\omega |S(j\omega)|$ represents then also a good evaluation of the performance rejection of the load disturbance. Finally, in order to achieve good transient response, good stability margin, good robustness in the face of model uncertainties and good rejection of the load disturbance, it is necessary to determine the parameters k_p, k_i and k_d such that:

$$\begin{cases} \zeta \geqslant \zeta_m \\ \max\limits_{k_p,k_i,k_d} \left\{ \min\limits_{\omega} \left| \dfrac{1}{S(j\omega,k_p,k_i,k_d)} \right| \right\} \end{cases} \tag{16}$$

There is not a known analytical solution of this optimization problem. A way to solve this problem is the numerical optimization. In the following, specific numerical optimization methods are proposed for the PI/PID controller for the plant models (1), (2).

3.1.1 Numerical optimization of the PI controller with the first-order plus dead-time process model

Consider the standard PI controller (4.1) and the process model (1), the open-loop transfer function is given by :

$$L(s) = \frac{k(1 + k_p T_i s)e^{-t_0 s}}{T_i s(1 + \tau s)}$$

with $T_i = 1/k_i$. Using the approximation $e^{-t_0 s} \approx 1/(1 + t_0 s)$, the polynomial characteristic of the closed-loop system is given by :

$$\rho(s) = s^3 + \frac{t_0 + \tau}{t_0 \tau} s^2 + \frac{1 + k_p k}{t_0 \tau} s + \frac{k}{T_i t_0 \tau}$$

which is of the form :

$$\rho(s) = (s + a)(s^2 + 2\zeta\omega_0 s + \omega_0^2)$$

with :

$$\begin{cases} a = & \dfrac{t_0 + \tau}{t_0 \tau} - 2\zeta\omega_0 \\ k_p = & \dfrac{(\omega_0 + 2a\zeta)\omega_0 t_o \tau - 1}{k} \\ k_i = & \dfrac{a\omega_0^2 t_o \tau}{k} \end{cases} \tag{17}$$

The closed-loop stability impose $a > 0$ which is verified if:

$$\frac{t_0 + \tau}{\zeta\omega_0 t_0 \tau} > 2 \tag{18}$$

The above inequality is satisfied for:

$$\frac{t_0 + \tau}{\zeta\omega_0 t_0 \tau} = b$$

with $b > 2$. Taking into account the first constraint of (16) one can choose $\zeta = \zeta_m$ which gives:

$$\begin{cases} \omega_0 = \dfrac{t_0 + \tau}{b\zeta_m t_0 \tau} \\ a = \dfrac{t_0 + \tau}{t_0 \tau} - 2\zeta_m \omega_0 \end{cases} \tag{19}$$

The optimization problem is then written as follows:

$$\begin{cases} \max\limits_{b>2} \left\{ \min\limits_{\omega} |1 + L(j\omega, b)| \right\} \\ L(s) = \dfrac{k(1 + k_p T_i s)e^{-t_0 s}}{T_i s(1 + \tau s)} \\ \omega_0 = \dfrac{t_0 + \tau}{b\zeta_m t_0 \tau} \\ a = \dfrac{t_0 + \tau}{t_0 \tau} - 2\zeta_m \omega_0 \\ k_p = \dfrac{(\omega_0 + 2a\zeta_m)\omega_0 t_o \tau - 1}{k} \\ k_i = \dfrac{a\omega_0^2 t_o \tau}{k} \end{cases} \tag{20}$$

which is numerically easy to solve. The PI controller is sufficient when the process dynamics is essentially first order. For higher-order processes the PI controller is not performing well, in this case the PID controller will be used (Ho & al. 1995).

3.1.2 Numerical optimization of the PID controller with the first-order plus dead-time process model

The dynamic performance obtained with the PI controller can be improved by the use of a PID controller. Consider the standard PID controller (4.1) and the process model (2), the open-loop transfer function is then given by:

$$L(s) = \frac{k(1 + k_p T_i s + k_d T_i s^2)e^{-t_0 s}}{T_i s(1 + \tau s)}$$

with $T_i = 1/k_i$. Using now the approximation $e^{-t_0 s} \approx 1/(1 + \frac{t_0}{2}s)^2$, the polynomial characteristic of the closed-loop system is given by:

$$\rho(s) = s^4 + \frac{t_0 + 4\tau}{t_0 \tau}s^3 + \frac{4(t_0 + \tau + k_d k)}{t_0^2 \tau}s^2 + \frac{4(1 + k_p k)}{t_0^2 \tau}s + \frac{4k}{T_i t_0^2 \tau}$$

in order to obtain an entirely adjustable polynomial characteristic, $\rho(s)$ can be put in the form:

$$\rho(s) = (s + a)^2(s^2 + 2\zeta\omega_0 s + \omega_0^2)$$

with :

$$\begin{cases} a = & \dfrac{4\tau + t_0}{2t_0\tau} - \zeta\omega_0 \\ k_p = & \dfrac{(a\zeta + \omega_0)a\omega_0 t_o^2\tau - 2}{2k} \\ k_i = & \dfrac{a^2\omega_0^2 t_o^2\tau}{4k} \\ k_d = & \dfrac{(a^2 + 4a\zeta\omega_0 + \omega_0^2)t_0^2\tau - 4(t_o + \tau)}{4k} \end{cases} \tag{21}$$

The closed-loop stability impose $a > 0$ which is verified if:

$$\frac{4\tau + t_0}{2\zeta\omega_0 t_0\tau} > 1 \tag{22}$$

The above inequality is satisfied for:

$$\frac{4\tau + t_0}{2\zeta\omega_0 t_0\tau} = b$$

with $b > 1$. Taking into account the first constraint of (16) one can choose $\zeta = \zeta_m$ which gives:

$$\begin{cases} \omega_0 = \dfrac{4\tau + t_0}{2b\zeta_m t_0\tau} \\ a = \dfrac{4\tau + t_0}{2t_0\tau} - \zeta_m\omega_0 \end{cases} \tag{23}$$

The optimization problem is then written as follows:

$$\begin{cases} \max\limits_{b>1} \left\{ \min\limits_{\omega} |1 + L(j\omega,b)| \right\} \\ L(s) = \dfrac{k(1 + k_pT_is + k_dT_is^2)e^{-t_0 s}}{T_is(1 + \tau s)} \\ \omega_0 = \dfrac{4\tau + t_0}{2b\zeta_m t_0\tau} \\ a = \dfrac{4\tau + t_0}{2t_0\tau} - \zeta_m\omega_0 \\ k_p = \dfrac{(a\zeta_m + \omega_0)a\omega_0 t_o^2\tau - 2}{2k} \\ k_i = \dfrac{a^2\omega_0^2 t_o^2\tau}{4k} \\ k_d = \dfrac{(a^2 + 4a\zeta_m\omega_0 + \omega_0^2)t_0^2\tau - 4(t_o + \tau)}{4k} \end{cases} \tag{24}$$

3.1.3 Numerical optimization of the PID controller with the second-order plus dead-time process model

The methods presented above can be extended for the second-order plus dead-time process model, the open-loop transfer function is then given by :

$$L(s) = \frac{k(1 + k_p T_i s + k_d T_i s^2) e^{-t_0 s}}{T_i s(s^2 + a_1 s + a_0)}$$

with $T_i = 1/k_i$. Using the approximation $e^{-t_0 s} \approx 1/(1 + t_0 s)$, the polynomial characteristic of the closed-loop system is now given by :

$$\begin{aligned} \rho(s) = \quad & s^4 + \left(a_1 + \frac{1}{t_0}\right) s^3 + \left(a_0 + \frac{a_1 + k_d k}{t_0}\right) s^2 \\ & + \frac{a_0 + k_p k}{t_0} s + \frac{k}{T_i t_0} \end{aligned}$$

which can be put in the form :

$$\rho(s) = (s + a)^2 (s^2 + 2\zeta\omega_0 s + \omega_0^2)$$

with:

$$\left\{ \begin{aligned} a = & \quad \frac{1}{2} a_1 + \frac{1}{2t_0} - \zeta\omega_0 \\ k_p = & \quad \frac{2(a\zeta + \omega_0) a\omega_0 t_0 - a_0}{k} \\ k_i = & \quad \frac{a^2 \omega_0^2 t_o}{k} \\ k_d = & \quad \frac{(a^2 + 4a\zeta\omega_0 + \omega_0^2 - a_0) t_0 - a_1}{k} \end{aligned} \right. \tag{25}$$

The closed-loop stability impose $a > 0$ which is verified if:

$$\frac{1}{\zeta\omega_0} \left(\frac{1}{2} a_1 + \frac{1}{2t_0} \right) > 1 \tag{26}$$

The above inequality is satisfied for :

$$\frac{1}{\zeta\omega_0} \left(\frac{1}{2} a_1 + \frac{1}{2t_0} \right) = b$$

with $b > 10$. Taking into account the first constraint of (16) one can choose $\zeta = \zeta_m$ which gives

$$\left\{ \begin{aligned} \omega_0 &= \frac{1}{\zeta_m b} \left(\frac{1}{2} a_1 + \frac{1}{2t_0} \right) \\ a &= \frac{1}{2} a_1 + \frac{1}{2t_0} - \zeta_m \omega_0 \end{aligned} \right. \tag{27}$$

The optimization problem is then written as follows:

$$\begin{cases} \max_{b>1} \left\{ \min_{\omega} |1 + L(j\omega,b)| \right\} \\ L(s) = \dfrac{k(1 + k_p T_i s + k_d T_i s^2) e^{-t_0 s}}{T_i s(s^2 + a_1 s + a_0)} \\ \omega_0 = \dfrac{1}{\zeta_m b} \left(\dfrac{1}{2} a_1 + \dfrac{1}{2t_0} \right) \\ a = \dfrac{1}{2} a_1 + \dfrac{1}{2t_0} - \zeta_m \omega_0 \\ k_p = \dfrac{2(a\zeta_m + \omega_0) a\omega_0 t_0 - a_0}{k} \\ k_i = \dfrac{a^2 \omega_0^2 t_o}{k} \\ k_d = \dfrac{(a^2 + 4a\zeta_m \omega_0 + \omega_0^2 - a_0) t_0 - a_1}{k} \end{cases} \tag{28}$$

4 Results

In this section various examples are presented to illustrate the proposed robust PI/PID controller design method.

4.1 Example 1

Consider the first-order plus dead-time model : $\frac{e^{-s}}{s+1}$. The proposed tuning method gives the following PI/PID controllers parameters ($\zeta_m = 0.7$) :

$$\begin{cases} \text{PI}: & k_p = 0.646 \quad k_i = 0.5712 \\ \text{PID}: & k_p = 0.846 \quad k_i = 0.7007 \quad k_d = 0.2501 \end{cases}$$

For comparison, simulation results are presented for the PI controller tuned by the Gain and Phase Margin (GPM) method (Ho & al., 1995). The GPM-PI controller parameters are : $k_p = 0.52$, $k_i = 0.52$ ($A_m = 3$, $\Phi_m = 60$).

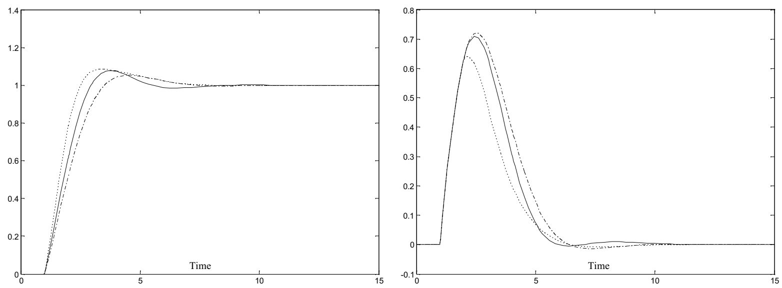

FIG. 2 – *Unit step response and load-distrbance response. GPM-PI controller (-.-), proposed PI controller (-) and proposed PID controler (...).*

Comparison results are shown in figure 2 for unit step response and load-disturbance response respectively. It is observed that the performance of the proposed PI/PID controller is better than that of GPM-PID controller.

For comparison with IMC-PI controller, consider the lag dominant first-order plus dead-time model $\frac{e^{-0.1s}}{1+s}$. The proposed tuning method gives the following PI controllers parameters ($\zeta_m = 0.5$): $k_p = 0.86$, $k_i = 2.66$. The corresponding parameters of the IMC-PI controller are (Astrom & Hagglünd, 1995): $k_p = 0.5$, $k_i = 2$.

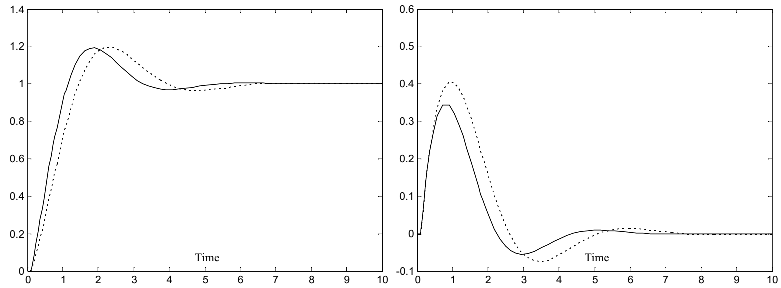

FIG. 3 – *Unit step response and load-distrbance response. IMC-PI controller ($\cdots$), proposed PI controller (-).*

Examples of responses to step change in set point and load-disturbance are shown in figure 3. The performance of the proposed PI controller is superior to that of IMC-PI controller.

Consider now, the non-minimum phase zero process model : $\frac{1-s}{(s+1)^3}$. The model used for the designing PID controller is : $\frac{e^{-1.58s}}{(s+1)^2}$. The proposed method gives the following PID controllers parameters : $k_p = 0.7983$, $k_i = 0.3514$, $k_d = 0.4497$ ($\zeta_m in = 0.75$). With the Gain and Phase Margin method the parameters are $k_p = 0.66$, $k_i = 0.33$, $k_d = 0.33$ ($A_m = 3$, $\Phi_m = 60$).

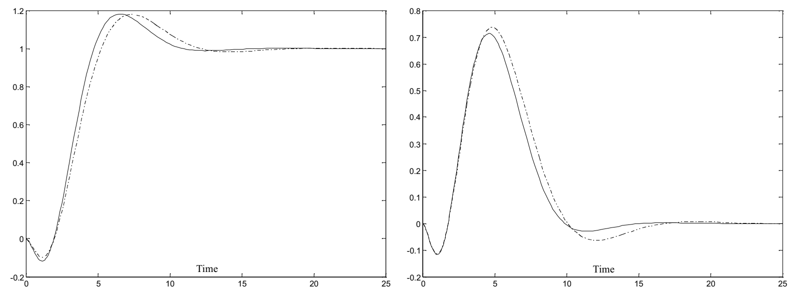

FIG. 4 – *Unit step response and load-distrbance response. GPM-PID controller (-.-), proposed PID controller (-).*

Comparison results are shown in figure 4 for unit step response and load-disturbance response repectively. Globally, the performance of the proposed PID controller is superior to that of GPM-PID controller.

4.2 Example 2

Consider the model of a stirred tank reactor :

$$\begin{aligned} \dot{C}_A = & \ \frac{q}{V}(C_{Af} - C_A) - k_0 C_A e^{-\frac{E}{RT}} \\ \dot{T} = & \ \frac{q}{V}(T_f - T) - \frac{\Delta H k_0}{\rho C_p} C_A e^{-\frac{E}{RT}} \\ & + \frac{\rho_c C_{pc}}{\rho C_p V} q_c (1 - e^{-\frac{h_A}{\rho_c C_{pc} q_c}} (T_{cf} - T) \end{aligned} \quad (29)$$

whose variables, parameters and nominal values are the same as defined in (Ge & al., 2002) and reproduced below.

Parameter	Notation	Value
Process flow rate	q	100 l/min
Feed concentration	C_{Af}	1 mol/l
Feed temperature	T_f	350 K
Coolant inlet temperature	T_{cf}	350 K
Reactor volume	V	100 l
Heat transfer coefficient	h_A	7×10^5 cal/min/K
Reaction rate constant	k_0	7.2×10^{10} min^{-1}
Activation energy term	E/R	1×10^4 K
Heat of reaction	ΔH	-2×10^5 cal/mol
Liquid densities	$\rho_,$, ρ_c	1×10^3 g/l
Specific heat	C_p, C_{pc}	1 cal/g/K

In this example, C_A is the measured output, q_c is the control variable and C_{Af} is the disturbance. Consider the stable region $C_A \in [0.06 \quad 0.14]$, the proposed synthesis method was applied in the worst-case of this domain, that is for $C_A = 0.14$ which gives undamped modes. The PID controller parameters are obtained as $k_p = 698.1$, $k_i = 1126{,}6$, $k_d = 367.1$. For comparison, simulation results are presented for the PID controller given by (Ge & al., 2002), which use a LMI approach for the synthesis. The LMI-PID controller parameters are $k_p = 516.6$, $k_i = 765.5$, $k_d = 143.8$.

Comparison results are shown in figure 5 for successive step change in the effluent concentration C_A that varies between 0.06 and 0.3. It is observed that in the whole operating regimes ($C_A \in [0.06 \quad 0.14]$), the performances of both PID controllers are similar. However for successive step in C_A between 0.15 and 0.3 (in this domain the process is unstable), the LMI PID control system becomes unstable while the proposed PID control system still remains stable. Figure 6 presents the disturbance response for three operating points. It is observed that the performance of the proposed PID controller is better than that of LMI-PID controller.

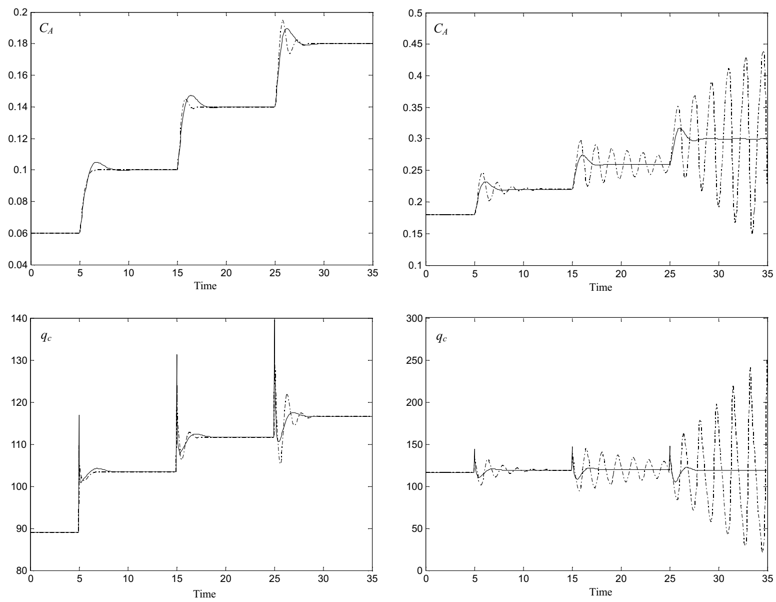

FIG. 5 – *Closed-loop responses to sucessive step changes in set point. LMI-PID controller (-.-), proposed PID controller (-).*

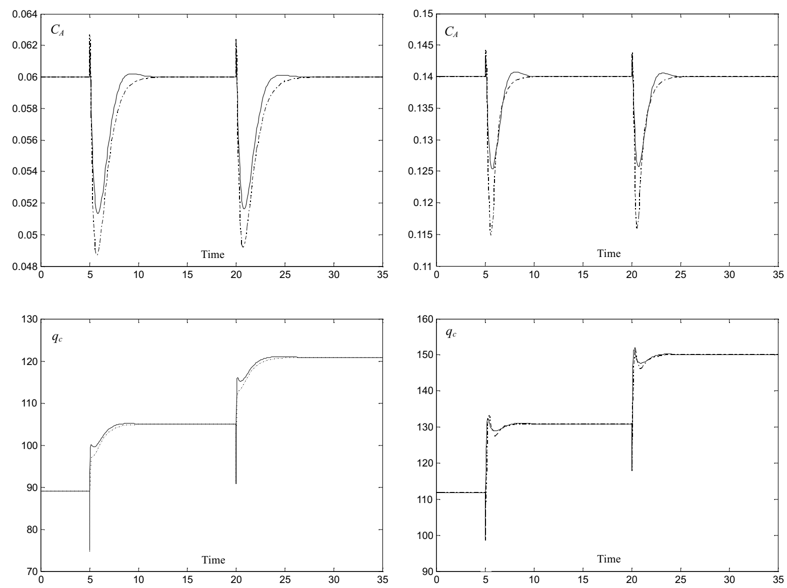

FIG. 6 – *Closed-loop responses to disturbance. LMI-PID controller (-.-), proposed PID controller (-).*

5 Conclusion

In this paper, a simple robust PI/PID controller design method was developed via numerical optimization approach. Various simulation studies have demonstrated the effectiveness of the proposed approach.

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