Robust Structured Controllers for Uncertain Piezoelectric Microactuators

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Abstract. This paper introduces a new control scheme which incorporate the concept of shaping filter together with the use of the ν-gap metric and the robust design of a structured controller. The main motivation in doing this is related to the development of efficient control laws for piezoelectric microactuators in the context of micromanipulation and microassembly. These systems are intrinsically nonlinear and very sensitive to the environmental conditions, thus requiring robust control to ensure a precise positioning despite disturbances. Designing a structured controller is known to be computationally intractable via the traditional $\mathbf{H}_\infty$ method. This is mainly due to the non-convexity of the resulting control problem which is of fixed order or structure type. To solve this kind of control problem easily and directly, without using any complicated mathematical manipulations and without using too many "user defined" parameters, we utilize the heuristic Kalman algorithm (HKA) for the resolution of the underlying constrained non-convex optimization problem. The experimental results validate the proposed technique and demonstrate its convenience for the development of fast and precise micropositioning systems.

Keywords: Piezoelectric microactuators, structured controller, $\mathbf{H}_\infty$ control, uncertain systems, non-convex optimization problems, heuristic Kalman algorithm.

1 Introduction

Piezoelectric materials are commonly used to develop systems acting at the micro- or nano-scale. This recognition is due to the high resolution, high bandwidth and high force density that they can offer [8]. They can be divided into two main categories as actuators: continuous (benders, stack, tube, etc.) or discrete (stickslip, inch-worm etc.). One of the applications of piezoelectric materials is the piezogripper which is used to perform micromanipulation/microassembly of small parts, such as biological objects or microstructures [17].

A piezogripper is made up of two piezoelectric cantilevers. When the cantilevers are subjected to electric voltage, they bend and can maintain and/or position a manipulated micro-object. If properly controlled, piezogrippers can pick, transport and place micro-objects with micrometric or submicrometric positioning accuracy. Such a precise positioning can be effectively achieved through the use of a feedback controller, which, if properly designed, results in a closed-loop system relatively insensitive to the external disturbances and modeling errors. This is an essential property for applications requiring micrometric positioning accuracy. Consequently, developing controller tuning techniques satisfying a set of imposed performance specifications are of great practical importance.

Broadly speaking, the various approaches dedicated to the control of piezoelectric actuators can be divided into three family of methods: those that take explicitly into account the non linearity of the system, mainly the hysteresis, those that control the system in such a way that the non linearity is nearly cancelled, and those that utilize a linear or a set of linear approximations of the system.

In the first family of methods the majority of the proposed approaches are based on feedforward hysteresis compensation (see for instance [12], [16], [15], [6] and references therein). The principle is to control the system through an inverse hysteresis operator in order to cancel the nonlinearity. This inverse compensation is never perfect and thus a feedback control is employed to minimize the disturbances effects.

In the second family of methods, the main proposed approaches consist in the use of a charge control instead of a voltage control [4], [14]. The relation between the electrical charge applied to the piezoelectric actuator and its displacement is indeed linear (the hysteresis effect is strongly reduced). Therefore, classical control techniques can be directly applied in a charge control scheme. However, the main difficulty with this approach resides in driving highly capacitive loads with available charge/current amplifiers [6].

In the third family of methods, the behaviour of the piezoelectric actuators is captured trough an uncertain model, which is typically a set of linear representation of the system. Piezoelectric cantilevers are indeed affected by ferroelectric nonlinearities such as hysteresis and creep and are also very sensitive to environmental conditions such as temperature, vibration etc. As a consequence, their static actuation characteristic is nonlinear and also their dynamical behavior may vary during operation. These variations cannot be taken into account via an LTI model, therefore a controller designed with a simple LTI model will work poorly or even become unstable. A powerful way to tackle this kind of difficulty is to consider not just a single LTI model but a family of LTI models that can account for the various dynamical behaviors observed on the real system [24]. This principle leads to the notion of uncertain linear system, which can be defined as the set $\mathcal{G} = \{G(\Delta) : \Delta \in \mathcal{D}_\Delta\}$ where $G(.)$ is a linear operator which associates an output vector to a given input vector. This operator is parametrized by the uncertainty block $\Delta \in \mathcal{D}_\Delta$, where $\mathcal{D}_\Delta$ can be a set of parameter vectors reflecting our ignorance about the system parameters. and also a set of stable systems allowing to take into account the neglected dynamics. Thus, for every $\Delta \in \mathcal{D}_\Delta$, we have a particular system model denoted $G(\Delta)$. The controller design is then based on the uncertain model $\mathcal{G}$ of the system. This approach allows obtaining controllers that can have acceptable performance while maintaining a suitable level of robustness [19], [10], see also [6] and references therein.

In these three family of methods, the problem of designing a suitable controller for the piezoelectric actuator arises. To this end many techniques have been used including adaptive control, sliding mode control, gain scheduling, robust control etc, see the survey [6] for a more detailed study.

The method presented in this paper belongs to the third family of methods and we focus mainly on the robust controller design problem which consists in finding a feedback controller ensuring the stability and acceptable performance for a given uncertain system $\mathcal{G}$. The traditional $\mathbf{H}_\infty$ control has been proved as a powerful way to solve the robust control problem [27]. However, the controller thus obtained is of high order. This is a serious limitation especially when the memory and computational power resources are limited, such in embedded controllers. Moreover, traditional robust control is unable to incorporate constraints into the structure of the controller. This is also a strong limitation especially when the control law must be implemented into commercially available controllers which usually have a fixed structure such as a PID [22], [7].

In this paper we introduce a new control scheme for piezoelectric actuator which incorporate the concept of shaping filter together with the use of the ν-gap metric and the robust design of a structured controller (PID type). The shaping filter can be seen as an open-loop controller designed to obtain a desired transfer function $G(s)$ chosen in accordance to the desired performance. However, since the system is only known through a set of models, the shaping filter is designed robustly in the sense that the distance measure (the ν-gap metric) between a member of the uncertain system $\mathcal{G}$ and the desired transfer function $G(s)$ is as small as possible. Then a robust feedback controller (PID type) is designed to minimize the disturbances effect. For solving the resulting nonconvex optimization problem we make use of the Heuristic Kalman Algorithm (HKA) [21], [23] because it requires only three "user defined" parameters. This is a major advantage over other stochastic optimization methods which require usually many more parameters to adjust (see [24] for a more detailed discussion about this point).

The remaining part of this paper is organized as follows. In Section 2, a general formulation of the robust structured control of piezoelectric microactuators represented by an uncertain parametric model is presented. This is done through the use of the *ν-gap metric* which is a measure of the closeness of two systems in terms of their closed-loop behavior. Section 3 describes the experimental setup used to validate the proposed design method. Experimental results show the efficiency of the designed structured controller. Finally, Section 4 concludes this paper.

2 Robust Control of Piezoelectric Cantilevers

In this section, the robust control problem of a piezoelectric microactuator is formulated as an optimization problem of the form given in [22], which can be solved via the Heristic Kalman Algorithm [23], [24].

The actuator considered in our experiments is a composite bimorph piezoelectric cantilever of a rectangular cross-section. This kind of actuator is realized with two layers: an active, piezoelectric layer (for instance Lead-Zirconate-Titanate, or PZT) and a passive layer (for instance Nickel). According to the reverse piezoelectric effect, a strain (contraction or expansion) results in the active layer under the application of an electrical field, which leads to a general deflection δ of the cantilever (see figure 1). The value of the tip deflection can then be accurately controlled

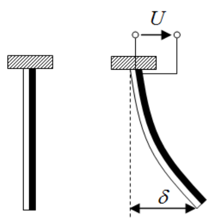

Figure 1: Principle of a piezoelectric actuator.

by an appropriate choice of the applied voltage. This can be done very efficiently through the use of a feedback controller. This aspect will be considered in Section 2.5 through the formulation of a robust control problem. Before that, we have to introduce some basic elements which are necessary for the robust control problem formulation.

2.1 System Model

The mathematical description of a real system involves some physical parameters whose values are uncertain. Accordingly, it will be assumed that the system to be controlled can be represented by an uncertain model defined by

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \mathcal{G} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \quad \mathcal{G} = \left\{ \tilde{G} = \begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} : \theta \in \Theta \right\} \tag{1}$$

where , $x \in \mathbf{R}^{n_x}$, $u \in \mathbf{R}^{n_u}$, $y \in \mathbf{R}^{n_y}$ are the state vector, the input vector and the output vector, respectively. The matrices $A(.)$, $B(.)$, $C(.)$ and $D(.)$ are of appropriate dimension and depend on the uncertain parameter vector θ. The parameter vector is not known a priori, the only knowledge we have is that it belongs to a certain bounded set Θ. Generally, the set of vector parameters Θ is defined as a hyper-rectangle in the parameter space, called the *parameter box*. This means that each uncertain or time-varying parameter θ_i belongs to a known interval *i.e.*, $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$, where the bounds $\underline{\theta}_i$ and $\bar{\theta}_i$ are assumed to be known.

2.2 Structured Controller

In the sequel, the notation $[\mathcal{G}, K]$ represents the closed-loop interconnection between the uncertain system $\mathcal{G}$, defined by (1) and the controller K. In our framework, K is a structured controller (e.g., a PID), which must be designed so that the closed-loop system $[\mathcal{G}, K]$ satisfies some desired performance. In the case of a MIMO PID controller, K matrix is given by

$$K(s) = K_P + K_I \frac{1}{s} + K_D \frac{s}{1 + \tau s} = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array} \right] = \left[\begin{array}{cc|c} 0 & 0 & K_I \\ 0 & -\frac{1}{\tau} I & -\frac{1}{\tau^2} K_D \\ \hline I & I & K_P + \frac{1}{\tau} K_D \end{array} \right] \tag{2}$$

where K_P is the proportional gain matrix, K_I and K_D are the integral and derivative gain matrices respectively, and τ is the time constant of the filter applied to the derivative action. This low-pass first-order filter ensures the properness of the PID controller and thus its physical realizability. We denote by $q = \text{vect}(K) = \text{vect}(K_P, K_I, K_D, \tau)$ the controller parameter vector. This vector belongs to the set $\mathcal{D}$ of possible settings for the controller, we have

$$q \in \mathcal{D} = \{ q \in \mathbf{R}^{n_q} : \underline{q} \preceq q \preceq \bar{q} \} \tag{3}$$

where the bounds $\underline{q}$ and $\bar{q}$ reflect the search domain over which the robust controller design problem is considered. A fundamental requirement of any control system is stability. Given the set of possible settings $\mathcal{D}$, we must retain only those ensuring the stability of the closed-loop system $[\mathcal{G},\, K]$. We denote by $\mathcal{K}_{\mathcal{G}}$ the set of stabilizing structured controllers of the uncertain system $\mathcal{G}$, we have: $\mathcal{K}_{\mathcal{G}} = \{q = \mathrm{vect}(K) \in \mathcal{D} : [\mathcal{G},\, K] \text{ is stable}\}$. This is a robust stability condition since a single controller must stabilize a set of plants.

2.3 Robust Stability/performance Test via $\nu-$Gap Metric

The *ν-gap metric* is a measure of the closeness of two systems in terms of their closed-loop behavior [26]. The ν-gap metric between two systems G_1 and G_2, denoted $\delta_\nu(G_1, G_2)$, is defined as

$$\sqrt{1-\delta_\nu(G_1,G_2)^2} = \left\| \begin{bmatrix} G_2 \\ I \end{bmatrix} (I + G_1^* G_2)^{-1} \begin{bmatrix} G_1^* & I \end{bmatrix} \right\|_\infty^{-1} \tag{4}$$

if $\eta([G_2,\, -G_1^*]) = \eta([G_1,\, -G_1^*])$, or $\delta_\nu(G_1, G_2) = 1$ otherwise. The notation $\eta(.)$ represents the number of right half plane poles of the system passed in argument and $[.\ ,\ .]$ represents the standard feedback interconnection of the systems passed in argument. Small values of $\delta_\nu(G_1, G_2)$ mean that the systems G_1 and G_2 are similar, thus we have $\delta_\nu(G_1, G_2) = 0$ only if $G_1 = G_2$. A remarkable property is that the generalized stability margin $b_{G,K}$ which will be defined in (6), and the uncertainty measure $\delta_\nu(G_1, G_2)$ are related by the following inequality (see [26])

$$\arcsin(b_{G_2,K}) \geq \arcsin(b_{G_1,K}) - \arcsin(\delta_\nu(G_1,G_2)) \Longrightarrow b_{G_2,K} \geq b_{G_1,K} - \delta_\nu(G_1,G_2) \tag{5}$$

where $b_{G,K}$ is the *generalized stability margin* defined as

$$b_{G,K} = \begin{cases} \left\| \begin{bmatrix} I \\ -K \end{bmatrix} (I + GK)^{-1} \begin{bmatrix} G & I \end{bmatrix} \right\|_\infty^{-1} & \text{if } [G,\, K] \text{ is stable,} \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

The quantity $b_{G,K}$ can be seen as a performance measure of the closed-loop system composed by G and K. High values of $b_{G,K}$ mean good performance of the closed-loop system. Therefore the inequality means that if a controller K performs well with the system G_1, and if the difference between G_1 and G_2 is sufficiently small, then K will also perform well with G_2. More precisely, according to (5), the closed-loop system $[G_2,\ K]$ is stable if $b_{G_2,K} > 0$, from (5), we can see that $b_{G_2,K} > 0$ if and only if $b_{G_1,K} > \delta_\nu(G_1, G_2)$. Therefore, if $b_{G_1,K} > \delta_\nu(G_1, G_2)$ then K, which is a stabilizing controller for G_1, is also a stabilizing controller for G_2. We then have the following result [26]. *If K is a stabilizing controller for a given system G, with $b_{G,K} = \alpha$, then K is also a stabilizing controller for any system $\tilde{G}$ satisfying $\delta_\nu(G, \tilde{G}) < \alpha$.* Note that the computation of

$\delta_\nu(G, \tilde{G})$ does not involve the expression of K (see relation 4), therefore, when trying to stabilize an uncertain system, we can test the above condition before calculating the controller. To ensure the robust stability/performance of the closed-loop system against parametric uncertainties, we have to determine the largest ν-gap metric between the nominal system and $\tilde{G} \in \mathcal{G}$. This can be done by solving the following maximization problem

$$\underset{\tilde{G} \in \mathcal{G}}{\text{maximize}} \quad \delta_\nu(G, \tilde{G}) \tag{7}$$

This non convex problem can be solved via the HKA presented in Section **??**. Note that (7) is independent of the controller and can then be solved before any design procedure.

Nominal System. Usually, the nominal model can be any member of the set $\mathcal{G}$. For instance, we can pick-up randomly a model out of the uncertain system $\mathcal{G}$ and declare it as the nominal model. As seen before, a controller $K(s)$ designed with one member of the set $\mathcal{G}$, is satisfying for any member of $\mathcal{G}$ if $b_{G,K} > \nu_{max}$, where ν_{max} is solution of (7). Thus we can see that the choice of the nominal model is unimportant provided that it belongs to $\mathcal{G}$. However, to simplify the design stage, it is often desirable to transform the original robust control problem into another, by an appropriate choice of a nominal model not member of $\mathcal{G}$. This can be done by shaping the uncertain system in such a way that it becomes close to the chosen nominal model. To be more precise, let G be the nominal model that we want to use for the design of the controller, and let $F(s)$ be the unknown shaping filter. The uncertain shaped system, denoted $\mathcal{G}F$ must be as close as possible to the nominal model G. This can be achieved in the ν-gap metric sense by finding the shaping filter F solution to the following optimization problem

$$\underset{F \in \mathcal{F} \subset \mathbf{RH}_\infty}{\text{minimize}} \ \max_{\tilde{G} \in \mathcal{G}} \delta_\nu(\tilde{G}F, G) \tag{8}$$

where $\mathbf{RH}_\infty$ is the set of proper and stable transfer matrices. The choice of the set $\mathcal{F}$ is problem dependant and is linked to the structure of the uncertain system $\mathcal{G}$. As illustrated in Section 3.1, the nominal model together with the set of shaping filters $\mathcal{F}$, can be determined through an inversion principle.

2.4 Performance Specifications

The desired performances of the closed-loop system are very often specified by some characteristics of the step response, namely the damping (overshoot), the settling time and the steady state error. For instance, a settling time of about 5 ms, a maximum steady state error of 0.1% and no overshoot are usual requirements for the piezoelectric microactuators used in precise positioning systems [10]. This kind of specifications can be enforced by appropriately shaping the largest singular value of the sensitivity function. This can be done as follows $\|WS\|_\infty \leq 1$, where S is the sensitivity function defined by $S = (I + GK)^{-1}$, and W is a frequency-dependent weighting functions, which is set in order to meet the performance specifications of the closed-loop system. The problem is then to find a stabilizing controller that satisfies this $\mathbf{H}_\infty$-norm constraint.

2.5 Formulation of the Robust Control Design Problem

We are now in position to formulate the robust control design problem. Given the uncertain system $\mathcal{G}$ defined by (1), a nominal model $G(s)$ which can be chosen as a member[1] of $\mathcal{G}$, a structured controller $K(s, q)$, where $q \in \mathcal{D}$ is the controller parameter vector and $\mathcal{D}$ the set of possible setting (see (2) and (3)), the problem is to find $q \in \mathcal{D}$ such that the uncertain closed-loop system $[\mathcal{G}, K]$ is stable while satisfying the desired performance specified by $\|WS\|_\infty \leq 1$, with a minimal decay rate. As seen in Section 2.3, the robust stability/performance is ensured if $b_{G,K}$ is bigger than the largest ν-gap metric in $\mathcal{G}$. Taking all that together, the controller parameter vector q ensuring the above requirements is the solution to the following optimization problem:

$$\begin{array}{ll} \text{minimize} & \max_i \mathrm{Re}(\lambda_i([G,\ K])) \\ \text{subject to:} & \left\| \begin{bmatrix} I \\ -K \end{bmatrix} (I + GK)^{-1} \begin{bmatrix} G & I \end{bmatrix} \right\|_\infty^{-1} \geq \max_{\tilde{G} \in \mathcal{G}} \delta_\nu(G, \tilde{G}) \\ & \|WS\|_\infty \leq 1, \quad q \in \mathcal{D} \end{array} \tag{9}$$

which is of the form given in [22] and can be solved with the HKA [24]. Note that $\max_{\tilde{G} \in \mathcal{G}} \delta_\nu(G, \tilde{G})$ does not involve K and can then be calculated before solving (9).

3 Presentation of the Experimental Setup

The proposed design method is now applied to the experimental setup shown in figure 2. The actuator is a composite bimorph piezoelectric cantilever of dimensions $l \times w \times h = 18.0 \times 2.0 \times 0.3$ mm (clamped region about 3 mm). The tip deflection of the piezoelectric cantilever, which is of about 0.8μm per volt, is measured by a Keyence non contact optical sensor with a resolution of about 10nm. A DSP card (type dSPACE DS1103) is used for the real time implementation

[1] Alternatively, the nominal model $G(s)$ can be chosen not in $\mathcal{G}$ through the use of a shaping filter (see Section 2.3).

of the controller. A high voltage amplifier is used to feed the piezoelectric actuator from the control signal delivered by the DSP card.

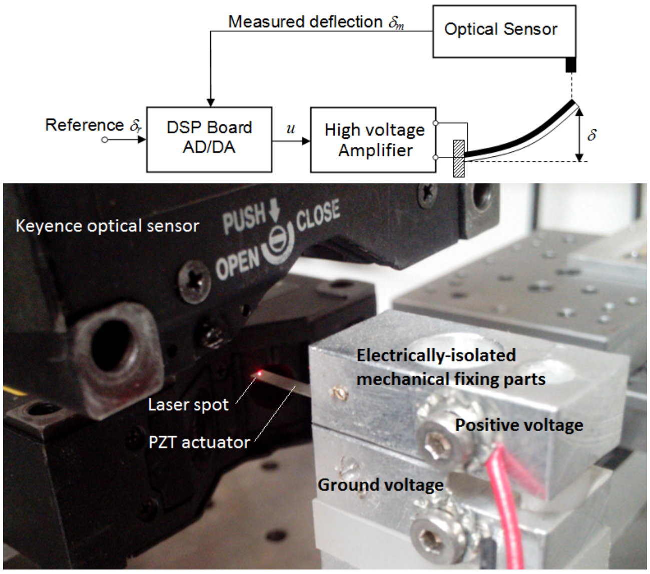

Figure 2: Block diagram and picture of the experimental setup.

3.1 Modeling and Identification

For a given operating point, the relation between the control input u and the output deflection at the tip of a piezoelectric cantilever can be modelled as in [15] by $\delta(s) = G(s)u(s)$, where $G(s)$ is the transfer function that must be identified via experimental data. We have found that a transfer function of the form $(b_1 s+b_0)e^{-t_0 s}/(s^2+a_1 s+a_0)$ fits well the measured data for various operating conditions. The dead-time t_0 accounts for the controller sampling time as well as for the sensory delays whose effects can not be neglected particularly for such fast settling times. To identify the parameters of the model, a succession of steps of amplitude ±20V around 0V, 40V and 80V has been applied to the piezoelectric actuator. As an example, Figure 3 shows the results obtained for a step response of amplitude 20V around 0V. In this figure the model $G_0^+(s)$ represents the transfer functions identified for a step input of amplitude +20V around 0V. It can be seen that the model $G_0^+(s)$ fits well the experimental data corresponding to the 20V step response. For each step response, the parameter vector $(a_0,\ a_1,\ b_0,\ b_1,\ b_2)^T$ has been evaluated via the output error method which is available in the Matlab System Identification Toolbox.

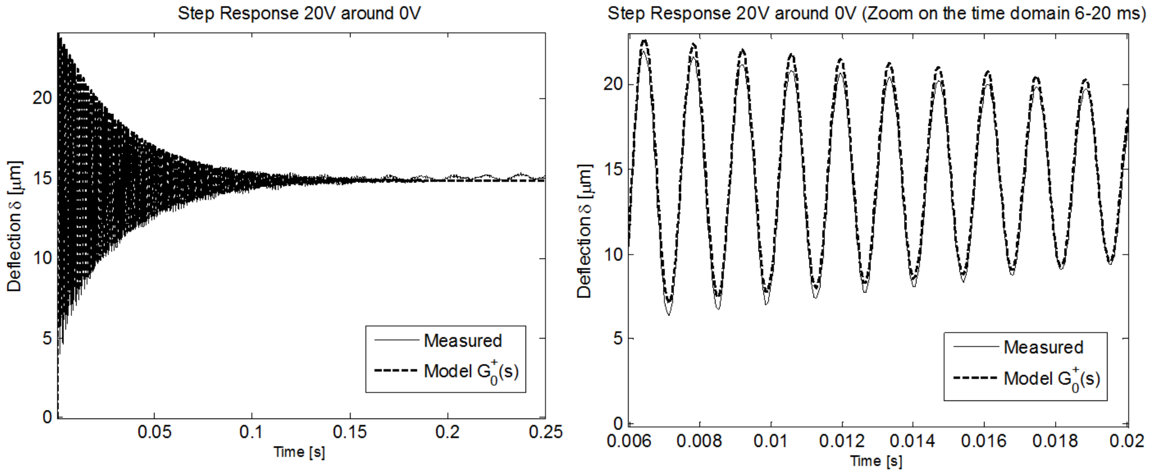

Figure 3: Step response of the piezoelectric actuator (amplitude 20V around 0V), compared with the step response of the model $G_0^+(s)$.

For the considered piezoelectric actuator, the following uncertain system can then be defined

$$\mathcal{G} = \left\{ G(s) = \frac{k(s+39850)}{s^2 + a_1 s + a_0} e^{-0.0002s} : \underline{\theta} \leq (k,\ a_0,\ a_1)^T \leq \bar{\theta} \right\} \tag{10}$$

where $\underline{\theta}$ and $\bar{\theta}$ are respectively the lower bound and the upper bound of the parameters of $G(s)$. These bounds can be determined as in [10] from the identified models. The bounds of the parametric uncertainties have been found to be $\underline{\theta} = (370,\ 2.01 \times 10^7,\ 50)^T$, and $\bar{\theta} = (450,\ 2.15 \times 10^7,\ 65)^T$. By solving the optimization problem (7) with HKA, we have found that the maximum of the ν-gap metric within $\mathcal{G}$ is less than 0.3. Thus, if a controller K is designed with a member of $\mathcal{G}$ such that $b_{G,K} > \nu_{\mathcal{G}}$ then this controller is stabilizing for the uncertain system $\mathcal{G}$.

Nominal Model. To design the controller, we have to define a nominal model for the system to be controlled. Although this can be done by taking a member of $\mathcal{G}$, we would like to illustrate the principle of the shaping filter presented in Section 2.3. To this end, assume that the nominal model is chosen according to the desired performances: settling time less than or equal to 5 ms, maximum steady state error less than or equal to 0.1%, no overshoot. In this respect, the nominal model can be taken as follows

$$G(s) = e^{-0.0002s}/((0.005/3)s + 1) \tag{11}$$

We have included the 0.2 ms dead-time introduced by the laser sensor in the system. This delay was causing closed-loop instability and could not be ignored in any way for high dynamic performance requirements. Now the objective is to determine a shaping filter $F(s)$ such that the ν-gap metric $\delta_\nu(\tilde{G}F, G)$ is as small as possible for any $\tilde{G} \in \mathcal{G}$ (see (8)). According to the structure of $\tilde{G}$ and the nominal model (11), the shaping filter can be chosen as

$$F(s) = \frac{s^2 + \alpha_1 s + \alpha_0}{\kappa(s + 39850)(\frac{0.005}{3}s + 1)} \tag{12}$$

Note that $F(s)$ is somehow a filtered version of the inverse of $\tilde{G}$. The parameters $(\kappa,\ \alpha_0, \alpha_1)^T$ of the shaping filter are determined in order to minimize the largest ν-gap metric between G and the uncertain shaped system $\mathcal{G}F$. This is done by solving the following optimization problem (see Section 2.3)

$$\begin{array}{ll} \text{minimize} & \max\limits_{\tilde{G} \in \mathcal{G}} \delta_\nu(\tilde{G}F, G) \\ \text{subject to} & F \in \mathcal{F} = \left\{ \dfrac{s^2 + \alpha_1 s + \alpha_0}{\kappa(s + 39850)(\frac{0.005}{3}s + 1)} : \underline{\theta} \leq (\kappa,\ \alpha_0,\ \alpha_1)^T \leq \bar{\theta} \right\} \end{array} \tag{13}$$

where the bounds $\underline{\theta}$ and $\bar{\theta}$ are the same as those of the uncertain system $\mathcal{G}$. Using the HKA, the following solution has been found: $\kappa = 444.67$, $\alpha_0 = 2.026 \times 10^7$, $\alpha_1 = 64.8$. The largest ν-gap metric between $G(s)$ and the uncertain shaped system $\mathcal{G}F$ has then been found to be less than 0.37. Consequently, if a controller K is designed with the nominal model G such that $b_{G,K} > 0.37$ then this controller is satisfactory for any member of the shaped system $\mathcal{G}F$.

3.2 Structured Controller Design

Given a nominal model of the piezoelectric actuator, the problem is to design a robust PID controller (2), so that the closed-loop system satisfies the performance specifications. As seen in Section 2.5, this can be done by solving the optimization problem (9). In what follows, after having introduced the weighting function on S, the PID controller is designed in the case of a nominal model $G \in \mathcal{G}$, and in the case where the nominal model is obtained via a shaping filter. Finally a comparison is done with the standard $\mathbf{H}_\infty$ control.

Weighting Functions. The controller must be designed to satisfy the following performance requirements: settling time less than or equal to 5 ms, maximum steady state error less than or equal to 0.1%, no overshoot. As seen in Section 2.4, these requirements can be satisfied by introducing a constraint on the sensitivity function of the form $\|W_1 S\|_\infty \leq 1$. The weighting function W_1 is chosen so that $1/|W_1(j\omega)|$ meets the performance requirement. One can check that this can be done via the following weighting function $W_1(s) = (s + 628)/(s + 0.628)$.

Controller Design via a Shaping Filter. We are now considering the uncertain shaped system $\mathcal{G}F$ where the shaping filter $F(s)$ is defined by (12). We can then make use of the nominal model (11) for the design of the PID controller. The transfer function (11) is approximated by a rational model using 2nd-order Padé approximation of the delays [25]. This lead to a 3th-order model. Problem (9) has been solved using HKA ($N = 50$, $N_\xi = 3$, and $\alpha = 0.3$) over the search domain $\mathcal{D}$ defined by $\underline{q} = [0\ 0\ 0\ 10^{-6}]^T$ and $\bar{q} = [10\ 10000\ 10\ 1]^T$, with $\max_{\tilde{G}\in\mathcal{G}} \delta_\nu(G, \tilde{G}) = 0.4$. The following controller parameters have been found

$$K_P = 3.82, \quad K_I = 4905, \quad K_D \cong 0 \tag{14}$$

Note that the resulting controller is of PI type. Figure 4 shows the nominal step response of the closed-loop system. We can see that the performance requirements are satisfied. Figure 4 also

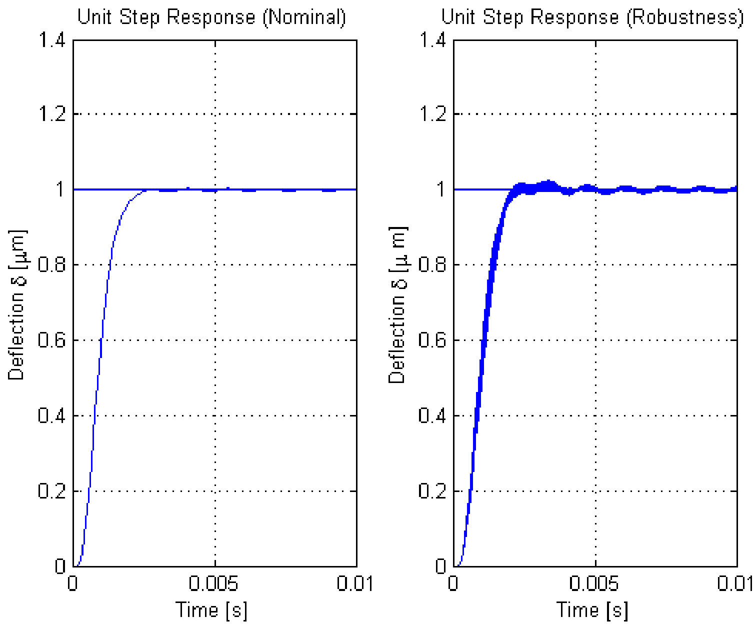

Figure 4: Simulations of the closed-loop response. On the left, the nominal case (system model (11)), on the right, the uncertain case obtained by taking various members of $\mathcal{G}F$.

shows the unit step response of the closed-loop system for various members of $\mathcal{G}$. This figure has been obtained in the same conditions as before. It can be observed that the closed-loop system is relatively insensitive to the considered parametric uncertainties. Note that even in the worst case, the settling time remains very small, less than 3ms. This result is twice better than the one obtained in [10], under very similar experimental conditions.

Controller Design via the Standard $\mathbf{H}_\infty$ Method. It is interesting to compare the preceding designs with the one achieved by the standard $\mathbf{H}_\infty$ sensitivity method. According to this method, a full order unstructured controller is obtained that minimizes $\|\mathcal{F}_l(P,K)\|_\infty$, where $\mathcal{F}_l(P,K)$ is the closed-loop transfer matrix and P is the augmented plant, *i.e.* including the weighting functions. The augmented plant can be calculated with the MatLab command `P=augw(G,W1,[],[])`, where `G` is the central model of $\mathcal{G}$ (*i.e.*, $\theta = (410,\ 2.08\times 10^7,\ 57.5)^T$), and `W1` the weighting function introduced in section 3.2. The controller K can then be obtained by using the MatLab function `hinfsyn` of the Robust Control Toolbox. This function solves coupled Riccati equations for designing the full-order $\mathbf{H}_\infty$-optimal controller. The use of the MatLab command `K=hinfsyn(P)` gives

$$\begin{aligned}
K(s) &= \frac{18393587.592 N_1(s) N_2(s)}{D_1(s) D_2(s)} \\
N_1(s) &= (s^2 + 57.5s + 2.01\times 10^7) \\
N_2(s) &= (s^2 + 3\times 10^4 s + 3\times 10^8) \\
D_1(s) &= (s + 1.316\times 10^6)(s + 3.985\times 10^4)(s + 0.2245) \\
D_2(s) &= (s^2 + 2.501\times 10^4 s + 3.216\times 10^9)
\end{aligned} \tag{15}$$

The closed-loop step response obtained with the controller (15) is shown in figure 5. It can be observed that in the worst-case, the settling time is about 6.5 ms. Note that the result obtained via the shaping filter approach is significantly better and, additionally, the structured controller is of low order.

3.3 Experimental Results

In this section the controller designed via the shaping filter approach is applied to the experimental set-up presented in figure 2. The controller has been implemented in a real-time DSP board with a sample frequency of 20 Khz.

Figure 6 shows the experimental results when a step reference δ_r of 20μm is applied to the system. The figure on the right is a zoom of the figure on the left for the time domain 0.015-0.035 s. The dotted line represents 0.25 times the control signal, in Volts, delivered by the high voltage amplifier. We can see that this experimental result is in good agreement with those obtained in simulation (see figure 4). The measured settling time is about 2.56 ms, which is much better than the result obtained in [10] for the same kind of piezoelectric actuators.

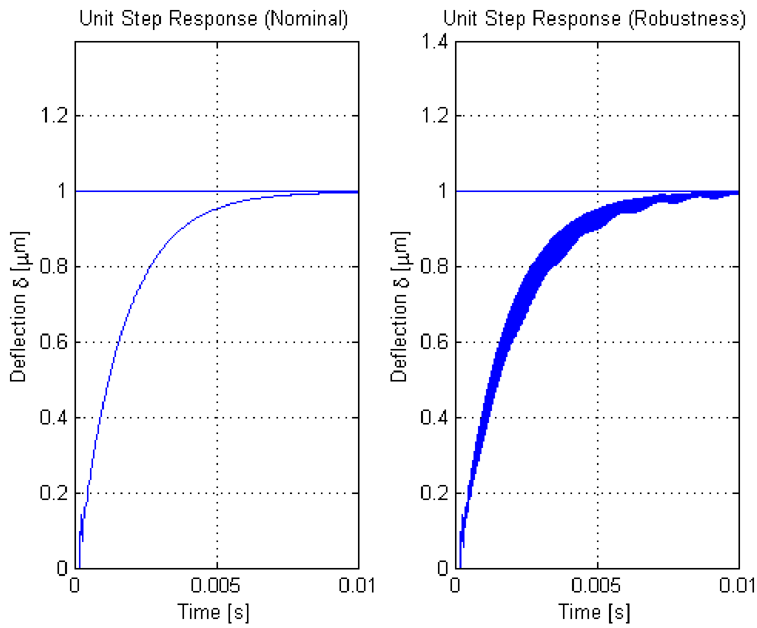

Figure 5: Step response of the $\mathbf{H}_\infty$ closed-loop system, in the nominal case and in the uncertain case (system model (10)).

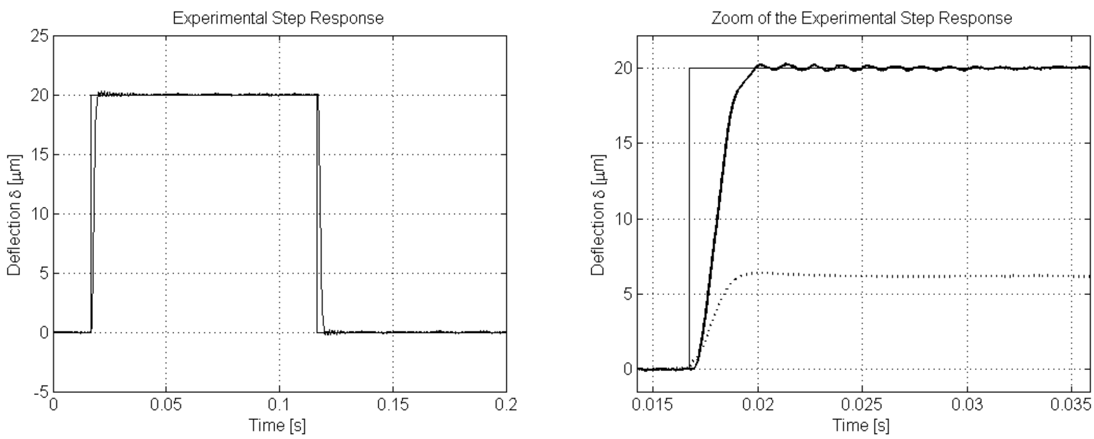

Figure 6: Experimental step response of the closed-loop system.

4 Conclusion

In this paper, the problem of designing a robust structured controller for uncertain piezoelectric microactuators has been investigated. This kind of system is indeed intrinsically nonlinear and very sensitive to the environmental conditions and thus requires a robust control to ensure a precise positioning despite the disturbances acting on the system.

To this end, we have introduced a new control scheme which incorporate the concept of shaping filter together with the use of the ν-gap metric and the robust design of a structured controller (PID type). The shaping filter is designed to obtain a desired transfer function $G(s)$ chosen in accordance to the desired performance. This has been done by minimizing the distance, in the ν-gap metric sense, between a member of the uncertain system $\mathcal{G}$ and the desired transfer function $G(s)$. A robust structured controller has then been designed to minimize the disturbances effect. This sort of control problem usually results in a non-convex constrained optimization problem which is known to be very difficult to deal with. This is why, to solve in a direct way this kind of control problem, we have proposed to use the Heuristic Kalman Algorithm. Indeed, HKA runs without any conservative assumption usually required in the conventional methods. In addition it allows to determine the controller parameters by solving the constrained optimization problem without requiring too many design parameters, unlike other stochastic algorithms.

The performance obtained with the proposed controller design method compare favourably to other approaches. This has been possible by the treatment of the dead time induced by the sensor as well as by using the concept of shaping filter. The experimental results validate the proposed technique and demonstrate its convenience for the development of fast and precise micropositioning devices.

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