Dynamic Reliability, Preventive Maintenance, Control of the Risk in Real Time

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SUMMARY

This article presents a concept of dynamic reliability based on the calculation of the probability of residual survival as evaluated in real time. This probability is calculated by taking into consideration the knowledge of the past operations scenario, revealed by means of the records of the stress sustained. This stress is evaluated with the help of the in-car information sources, sensors, measuring chains, and recorders. The whole of the information is used to define the parameters of our calculation model. In this model two methods of data processing are presented. If the first one memorizes the history of the operation's real conditions, the other one – based on the system's state function – naturally considers the past using the failure rate derivative. These approaches make it possible to make more realistic and precise decisions concerning the preventive change, which consequently leads to the following advantages: the extension of the period preceding the change, and a better control over the risk.

Key words: dynamic reliability / PNHP/ Cox pattern / the Weibull law / damage, preventive maintenance, predictive maintenance.

1. INTRODUCTION

As part of the studies of operation and risk control, the reliability evaluation assumes a particularly important character, in both the aspect of the risk (probability of lack of failure leading to a serious event), and that of the cost of the operation (the extension or reduction of the periods of preventive change). It is important to notice that the operating periods accepted for the vital components have been established on the basis of operating scenarios – evaluated a priori – showing a significant distortion, compared with the real use scenario. This difference leads to two defects, the first one concerning the risk control, the second one the operating costs. The actual knowledge of operating conditions can issue during the use, thanks to properly adapted technical systems (sensors, recorders, in-car measuring chains). This knowledge used in the calculations of residual reliability obtained during the operation facilitates a more precise estimation of the interest values. It is this approach that is implemented in our concept of dynamic reliability. A significant benefit can be proved for both of the aspects mentioned above, justified by the consideration of the operation's real history. Having a direct impact on the residual reliability, this history is taken into account in the calculation of dynamic reliability, suggested here. What is reached is a better estimation of the reliability entities and an optimisation of operating security parameters (Time Between Overhaul TBO). This concept is inspired with the studies of optimisation of preventive maintenance periodicities, maintenance prognostication, and optimisation of the actions following inspection. A number of works on the preventive maintenance has been suggested, optimisation of the maintenance inspections [ZEQ 06], [KOT 06], opportunist maintenance. One needs to mention the Xiaojun Zhou's [XIA 06] recent works on predictive maintenance based on the reliability. He suggests a sub-preventive model of failure rates, integrating an ageing that leads to a decrease of the preventive periodicities. Works by Chin Tai Chen [CHI 02] put forward dynamic preventive maintenance based on the Markov model for a system. Our suggestion is the continuation of these works as far as the aim is concerned (optimisation of the preventive maintenance actions, involving the knowledge of the real disrepairs). However, in terms of the calculation method, this «dynamic reliability» approach differs from all the preceding ones, as it is based on the updated evaluation of the reliability model, integrating the real operation scenario. This makes it innovative, and should lead to a better evaluation of the reliability in real time.

2. METHODOLOGY OF THE RELIABILITY EVALUATION IN REAL TIME

The quantitative evaluation of the reliability establishes itself on the basis of the shelf life models, most representative of the scenario of the operation envisaged during the designing. Yet, it seems better to remain pessimist and consider the most severe conditions, pertaining to at least 90 % of the cases. This «a priori» scenario, an essential data in the provisional evaluation, manifests itself by an extensive magnitude of time (number of cycles, duration of solicitation, etc…) and the stress conditions on the component (strain, temperature, degree of humidity, speed…). Consequently, the reliability model's parameters should be able to define themselves on the basis of this scenario. It is the case of the simple Weibull's model, of a Weibull's model associated with a Cox model, enabling the environmental stress to be integrated more easily. Sufficiently general, these representations cover a wide range of cases: they will be retained and used for the sake of the further study and demonstration. The methodology and calculations of this approach are founded on the development of:

1. a model of evaluation of the real damage sustained during the operation. This model takes into account the real operation scenario through the environmental parameters considered as relevant [BAG 01], [COX 72], [COX 75], [BOM 73].

2. a system of measuring and recording of the operation in real time. The real conditions are evaluated on the basis of diverse sources of information: temperature sensor, vibration sensor, real operating mode, speed, overrunning of normal operating conditions, [DEV 98]. These are the environmental parameters of the system.

3. a formulation of the residual reliability calculation, integrating the operating history. This will translate as a description of an algorithm integrating the relevant information of the system. A function of the multi-parameter reliability is developed, making it possible to modify the strategic decisions according to the accepted risks.

Hypotheses:

- The component has functioned without any failure until the moment *T*
- The operating history is known by means of the function Z(t), which is documented on the basis of the sources of information of the system.

Notations used:

Z(t) represents the history of the damage in the form of a vector

t: time
$b^T=(b_1, b_2 , b_3 , \ldots b_n)$

$Z^T(t)=(Z_1(t), Z_2(t), Z_3(t), \ldots Z_n(t))$

$\overline{Z}^T(t) = (\overline{Z}_1(t), \overline{Z}_2(t), \overline{Z}_2(t), \ldots\ldots\overline{Z}_i(t), \ldots\ldots\overline{Z}_n(t))$

$\overline{Z}(T)$ represents the average equivalent damage for the period «*T*»

T: the time of the past operation, with the conditions described by *Z(T)*

$\hat{Z}(T+t)$ represents the estimation of the damage for «T+t» in the form of a vector

$R_T(t)$: the residual reliability calculated in real time. It integrates the past operation until T.

$b^T=(b_1, b_2 , b_3 , \ldots b_n)$: vector of affecting parameters

Weib(β, η): the Weibull law with two parameters

$\lambda(t, Z_{(T+t)})$: failure rate integrating the operating history until T

2.1. Model of evaluation of the conditions of real damage sustained during the operation

First of all, it is important to remember that the probability of operation at T+t – the fault during the period T being known – is described by the following relations:

$P(T+t/T) = R_T(t)$, then:
(1)

$$R_T(t) = \frac{R(T+t)}{R(T)} \qquad (2)$$

2.2. System of measuring and recording of the operation in real time

The impact of the past operation is taken into consideration on the basis of the function *Z(t)*, allowing for the integration of the damage sustained in the calculation of the residual reliability of the component or system. This reliability will, in consequence, be evaluated at the time T+*t*. In order to carry out this evaluation, the following couple of actions need to be introduced:

- *Z(t)*: function describing the operating fault on the basis of the variables Z_i, representing the different operating environments (indicator of the stress sustained).

- $\lambda(t, Z(t))$: expression of the failure rate, the calculation integrating the damage *Z(t)*.

The history of the past operation is described on the basis of the indicators most representative of the failure modes sustained by the component or system. These damages are recorded thanks to the sensors or the in-car measuring chains.

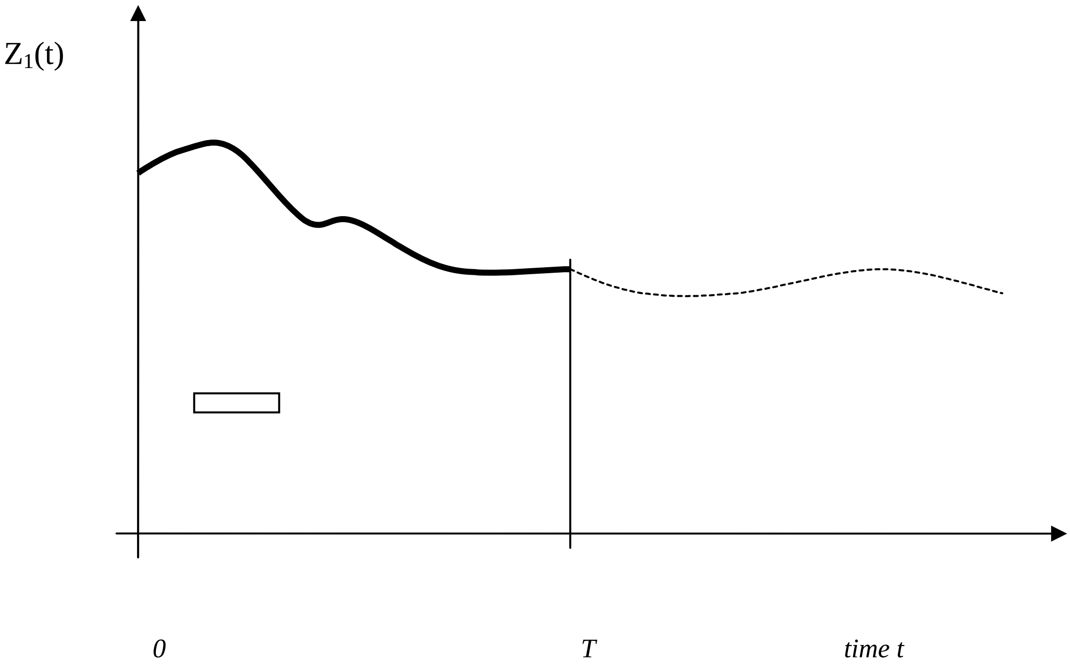

Diagram 1a: Indicator of the damage N°1 linked to the failure mode M1

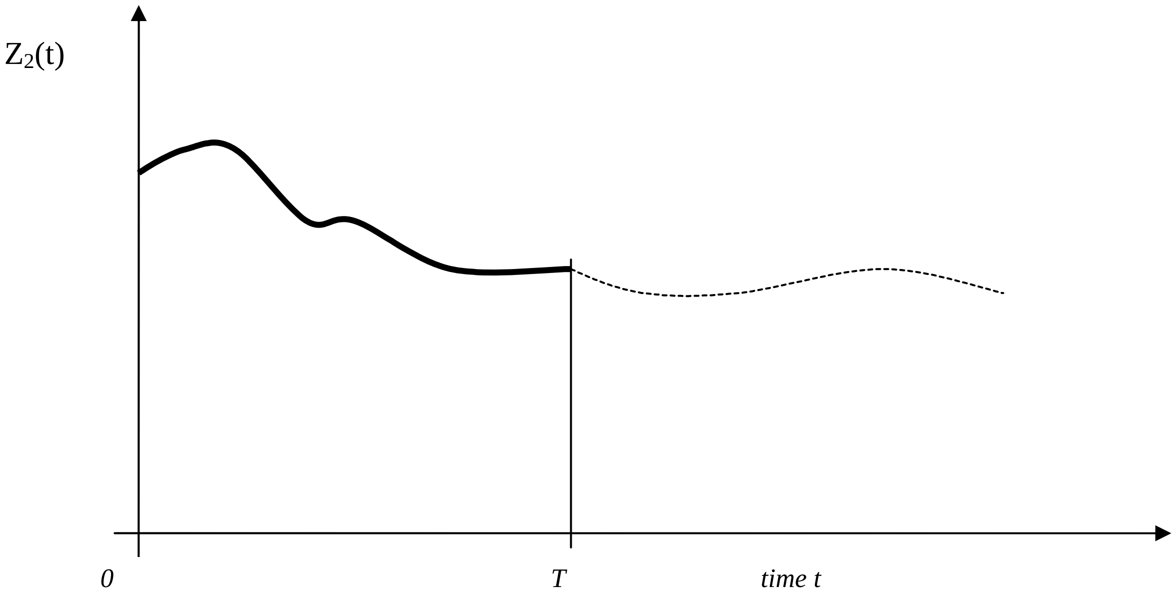

Diagram 1b: Indicator of the damage N°2 linked to the failure mode M2

2.3. Formulation of the residual reliability calculation, integrating the operating history

The description of this formulation can follow two ways:

1. A model allowing for the damage based on the failure rate with proportional hazard
2. An equation system constructed on the derivative of the failure rate and the state function equations

2.3.1. The model allowing for the damage based on the failure rate with proportional hazard

This function, relatively general, is defined by $\psi(Z(t))$, which can have diverse forms, the best representing the operating conditions. Within the framework of the encountered problems, it has seemed sufficient to use a proportional hazard function, i.e.:

$$\lambda(t, Z(t)) = \lambda_0(t) Z(t) \tag{3}$$

$$Z(t) = e^{b^T Z(t)} \tag{4}$$

hence:

$$\lambda(t, Z(t)) = \lambda_0(t) e^{b^T Z(t)} \tag{5}$$

The estimation of Z(t) for the predictive part on [T ; T+t] is done by: $\hat{Z}^T(t) = \overline{Z}^T(T)$

This is the simplest method that will be used for the sake of our presentation. It is necessary to notice that all the other, more sophisticated, prediction methods (exponential smoothing, regression, moving average) are able to be used if they appear to be justified.

In case of the use of the Cox model, the b_i parameters are evaluated on the basis of a classic experiences return. Their estimations are carried out using either the least squares method, or the maximal partial likelihood method. The real operation is obtained by means of the sensors, associated with, and then integrated in the calculation of the residual reliability, described by the following expression:

$$R(t/T) = R_T(t) = \frac{R(T+t)^{\exp\left(b^T\left(\frac{1}{T}\int_0^T Z(u)du + \frac{1}{t}\int_T^{T+t}\hat{Z}(u)du\right)\right)}}{R(T)^{\exp\left(b^T\left(\frac{1}{T}\int_0^T Z(u)du\right)\right)}} \tag{6}$$

In the case of the Weibull model, the following is reached:

$$R_T(t) = \frac{e^{-\left(\frac{T+t}{\eta}\right)^\beta \exp\left(b^T\left(\frac{1}{T}\int_0^T Z(u)du + \frac{1}{t}\int_T^{T+t}\hat{Z}(u)du\right)\right)}}{e^{-\left(\frac{T}{\eta}\right)^\beta \exp\left(b^T\left(\frac{1}{T}\int_0^T Z(u)du\right)\right)}} \tag{7}$$

2.3.2. The equation system constructed on the derivative of the failure rate and the state function equations

The present approach consists in extending the classic reliability methods to the dynamic case, taking into account the past damage, due to the past operating conditions, in which the process has evolved (see [] for more details). Therefore, the consideration of the history of evolutions possibly sustained by one of the system's component necessitates the elaboration of the memory of behaviour models. For that purpose, what we have suggested is the differential model of the failure rate, which, associated with the state function representation, makes it possible to reach this goal.

The equation of any dynamic state function can generally be presented as:

$$\begin{cases} \dot{x}(t) = f(x(t),u(t)) \\ x(t) = [x_1(t),\ldots,x_{n_x}(t)]^T, \quad u(t) = [u_1(t),\ldots,u_{n_u}(t)]^T \\ f(x(t),u(t)) = [f_1(x(t),u(t)),\ldots,f_{n_x}(x(t),u(t))]^T \end{cases} \tag{8}$$

where x stands for the state vector, u for the input vector, and f is a vector of functions, generally non linear, facilitating the characterization of the system's dynamics. A physical system, mathematically described by (8), corresponds to the assembling of a finite number of components. Our goal is to estimate in line the reliability of one or several components regarded as crucial for the security of the system's operation. As it has been noticed, it is necessary to take into account the operating conditions, because of their direct influence on the reliability. Naturally, the conditions of a particular component's operation depend on the entry u and on the state x, around which the system evolves. In these conditions, a realistic reliability model should include the variables of the physical processes, i.e. the input/state couple (u, x). Nevertheless, this is not sufficient, as the operating conditions change constantly in time. Therefore, it is equally necessary to take into account the history of the operating conditions in which the system has evolved. To that end, we suggest the use of a dynamic model of the failure rate, whose form is as follows:

$$\dot{\lambda}(t) = \phi(t, x(t), u(t)) \tag{9}$$

Where λ represents the failure rate, $\dot{\lambda}$ the derivative in respect to the time of the failure rate, and ϕ is a function of time and of the operating conditions, i.e. of the ($x(t)$,$u(t)$) couple.

The model (9) needs to be completed by the dynamic model (8), describing the evolution of the state under the action of the inputs. The complete model of dynamic failure rate has the following form:

$$\begin{cases} \dot{\lambda}(t) = \phi(t, x(t), u(t)) \\ \dot{x}(t) = f(x(t),u(t)) \end{cases} \tag{10}$$

A particular model, of a great practical importance, is as follows:

$$\begin{cases} \dot{\lambda}(t) = t^{n(x(t),u(t))}\psi(x(t),u(t)) \\ \dot{x}(t) = f(x(t),u(t)) \end{cases} \tag{11}$$

where ψ and n are unknown functions depending on the operating conditions $(x(t),u(t))$. For a constant operating condition, noted (x_0, u_0), the integration of $\dot{\lambda}$ in relation to the time leads to the following Weibull model:

$$\begin{cases} \lambda(t) = \dfrac{\beta(x_0,u_0)}{\eta(x_0,u_0)}\left(\dfrac{t}{\eta(x_0,u_0)}\right)^{\beta(x_0,u_0)-1} \\ f(x_0,u_0) = 0 \end{cases} \tag{12}$$

The relations linking the n and ψ functions to the Weibull's β et η parameters, for the operating condition (x_0, u_0) are then presented as follows:

$$\begin{cases} n(x_0,u_0) = \beta(x_0,u_0) - 2 \\ \psi(x_0,u_0) = \dfrac{\beta(x_0,u_0)(\beta(x_0,u_0)-1)}{\eta(x_0,u_0)^{\beta(x_0,u_0)}} \end{cases} \tag{13}$$

Using the Weibull's law's [Lyo 06] usual techniques of parametric transformation, the relations (13) make it possible to define point by point the unknown functions $\psi(x,u)$ and $n(x,u)$ for different operating points, i.e. for different points of the system's equilibrium. The model (11) thus obtained should allow for the realistic estimation of the failure rates in real time. Indeed, it enables the consideration of both present and passive environments, in which the particular component may have functioned. The principal difficulty of this approach consists in finding an adequate estimation of the unknown functions $\psi(x,u)$ and $n(x,u)$. It is in the next section that this problem will be discussed.

2.3.3. Approximation of the functions associated with the operating conditions

Let us consider l operating points (x_0^i, u_0^i), $i = 1,\ldots,l$. In order to simplify the notations, we note as follows: $\xi(t)$ is the condition of instant operation, i.e. $\xi(t) = [x(t) \quad u(t)]^T$, and ξ_i a particular operating condition: $\xi_i = [x_0^i \quad u_0^i]^T$. $\Xi = \{\xi_1, \xi_2, \ldots, \xi_l\}$ is the set of the operating conditions considered:

$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_l\} \tag{14}$$

For each operating condition, the histories of the particular component's service life are supposed to be known:

$$L_i = \{t_1^i, t_2^i, \ldots, t_{n_i}^i\} \tag{15}$$

These data can then be used to determine the parameters of the Weibull law for the operating condition characterised by ξ_i :

$$L_i \rightarrow (\beta(\xi_i), \eta(\xi_i)), \quad i = 1, \ldots, l \tag{16}$$

The details of the calculations revealing $\beta(\xi_i)$ and $\eta(\xi_i)$ on the basis of L_i can be consulted in [Lyo 06]. The relations (13) then allow for the determination of the value of the functions ψ and n for the operating conditions ξ_i . It needs to be observed that in this way, the functions $\psi(\xi(t))$ et $n(\xi(t))$ are only known for a finished number of points. Hence, it is possible, on the basis of the values $\psi(\xi_i)$ et $n(\xi_i)$, to construct a stepping approximation of the unknown functions $\psi(\xi(t))$ et $n(\xi(t))$, as presented in the diagram 16.

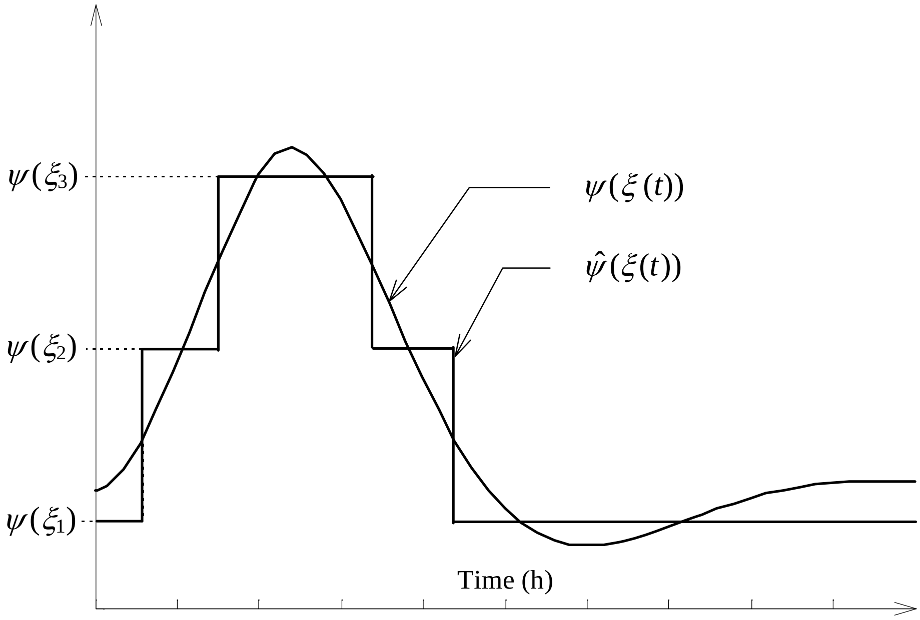

Diagram 2 – Stepping approximation of the function $\psi(\xi(t))$

This approximation can be constructed by determining $\xi \in \Xi$, the nearest point of the current operating condition $\xi(t)$. This is made possible by solving the following problem:

$$z(t) = \arg\min_{\xi_i \in \Xi} \left\| \xi(t) - \xi_i \right\| \tag{17}$$

where the symbol || || designates the Euclidean norm. In these conditions, we have $\psi(z(t)) = \psi(\xi_i)$ and $n(z(t)) = n(\xi_i)$, if $\xi(t)$ is next to ξ_i with $i = 1, \ldots, l$.

In this way, a function is obtained, which commutes with the different known values, in line with the operating condition observed.

The complete model of dynamic reliability assumes the following shape:

$$\begin{cases} \dot{\lambda}(t) = t^{n(z(t))}\psi(z(t)) \\ \dot{x}(t) = f(x(t), u(t)) \\ z(t) = \arg\min\limits_{\xi_i \in \Xi} \left\| \xi(t) - \xi_i \right\|, \quad \xi(t) = \begin{bmatrix} x(t) & u(t) \end{bmatrix}^T \\ \xi_i = \begin{bmatrix} x_0^i & u_0^i \end{bmatrix}^T, \quad f(x_0^i, u_0^i), \quad i = 1, \ldots l \end{cases} \tag{18}$$

Eventually, the failure rate at the moment *t*, taking into account the operating conditions varying in time, is given by:

$$\lambda(t) = \int_0^t \dot{\lambda}(\sigma) d\sigma \tag{19}$$

This information can be used to predict the reliability associated with a future operating condition given.
In fact, the reliability is expressed, according to the failure rate, by means of the relation:

$$R(t) = \exp\left(-\int_0^t \lambda(\sigma) d\sigma \right) \tag{20}$$

The prediction of the operating reliability can be defined as the conditional probability that the system will function at the future moment $t+\tau$, where t is the current time, $\tau \in [0, \tau_h]$ and τ_h the prediction horizon. This conditional probability expresses itself:

$$R(t+\tau|t) = \frac{R(t+\tau)}{R(t)} = \frac{\exp\left(-\int_0^{t+\tau} \lambda(\sigma) d\sigma \right)}{\exp\left(-\int_0^{t} \lambda(\sigma) d\sigma \right)} = \exp\left(-\int_t^{t+\tau} \lambda(\sigma) d\sigma \right) \tag{21}$$

It is worth noticing that $\lambda(t+\tau)$ is unknown for $\tau > 0$, but can be evaluated for a future operating condition given. In this case, there is:

$$R_i(t+\tau|t) = \exp\left(-\int_t^{t+\tau} \lambda_i(\sigma) d\sigma \right) \tag{22}$$

where λ_i is the failure rate associated with the operating condition number i ($i = 1, \ldots, l$).

3. VALIDATION AND APPLICATION

The method's validation, application, and illustration are developed on the basis of the study of a machine component having functioned during 250 hours on a vehicle, which represents a typical year. The environmental parameters are reduced to the minimum in order to simplify the reading. The value of the bi coefficient has been evaluated based on a history issued from the experience return. The simulation of these conditions is presented in appendix. As a result, the following calculation elements arise:

$\lambda(t, Z(t)) = \lambda_0(t)e^{b_1 Z_1(t)}$, with

$b_1=0,1785$

with Z(t) able to take 3 levels of constraints: $Z_1=0$, $Z_2=1$, $Z_3=2$

Weib (4,63; 217)

The evaluation of the residual reliability calculated after the time «t» chosen here, equal to 50 hours, is determined (cf. diagram 3). This evaluation is carried out using the «Matlab» software. The hypotheses established for the future environmental conditions are identical to those of the preceding range.

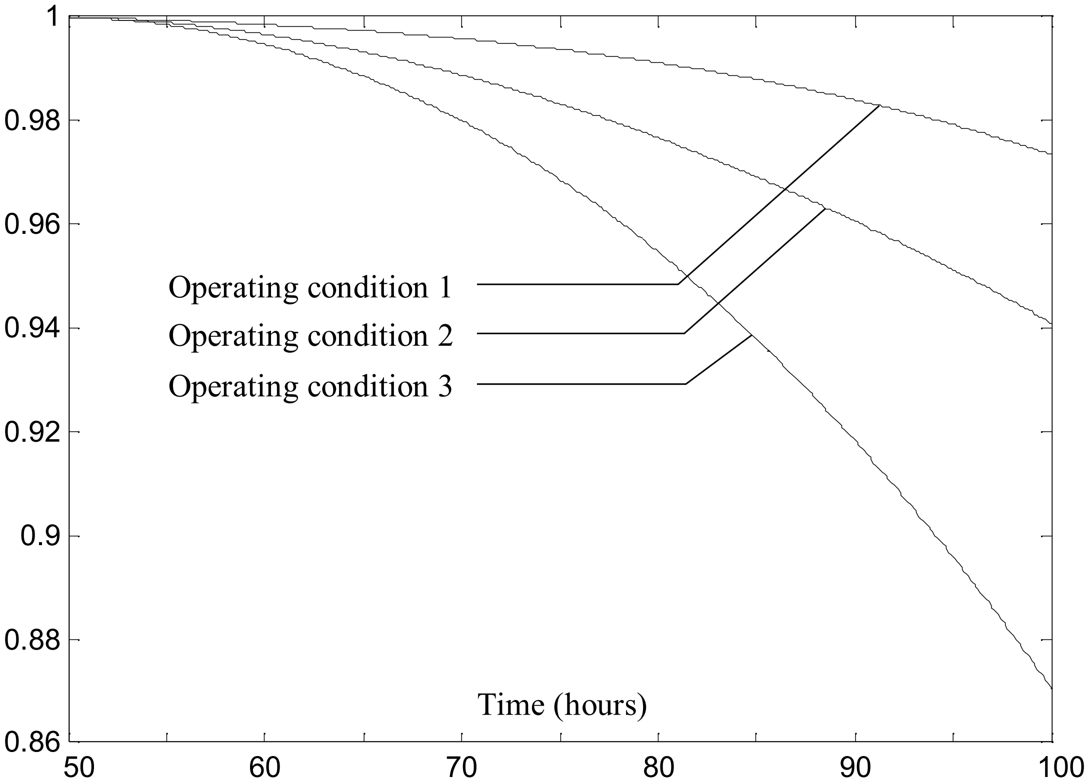

Diagram 3 – Reliability prediction on the basis of t = 50

The diagram 3 depicts the operating reliability prediction for the moment t = 50 on the prediction horizon of 50 hours, and for the three known operating conditions. On the basis of the figure 18's results, it is possible for example to see that the probability of the correct functioning of the system at t = 100h – knowing that it operated at t = 50h – is 0.885 for the operating condition 3. The observations of the same kind can be carried out for all the other operating conditions.

4. CONCLUSION

The usage of the concepts developed is promising, as are the applications of the preventive and deriving changes' optimisation. The systems' maintaining costs will be able to be reduced and better controlled. The risk will be better controlled as well. It is advised to implement the presented methods in in-car calculators, aeronautics, and automobile industry, among others.

APPENDIX 1: Recall of the Cox's partial likelihood

The contribution V_i of the failing component *i* in t_i with the partial likelihood V^* is equal to the conditional probability, whether it is the element *i* subjected to the constraints z_i and failing in t_i, knowing the whole of the population at risk at this moment. This leads to the following stages of the likelihood calculation, defined as [5], [1], [3]:

$$V_i = \frac{\lambda(t_i, z_i)}{\sum_{k \in nt_i} \lambda(t_i, z_k)}$$

$$V_i = \frac{\lambda(t_i, z_i)}{\sum_{k \in nt_i} \lambda(t_i, z_k)} = \frac{e^{b_i z_i}}{\sum_{k \in n(t_i)} e^{b_i z_k}}$$

$$V^* = \prod_{i=1}^{n} V_i$$

$$L^* = \ln(V^*)$$

$$L^* = \sum_{i=1}^{n} \left(b_i z_i - \ln\left(\sum_{j \in n(t_i)} e^{b_j z_j} \right) \right)$$

$$\frac{dL^*(\hat{b})}{db} = 0$$

APPENDIX 2. Simulation of a data base corresponding to the preceding operation

The simulation [7] considers a machine component that operated during 250 hours on a vehicle, which is representative of a typical operating year.

$T_{(i,Z,L1)}$	$T_{(i,Z,L2)}$	$T_{(i,Z,L3)}$	$T_{(i,Z,4)}$	$T_{(i,Z,L5)}$
238	633	498	222	175
246	450	249	359	139
231	606	396	87	92
103	563	643	154	210
166	403	334	66	73

It is on the basis of this type of procedure that the parameters have been estimated.

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BIOGRAPHY

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