Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ residual generator design via Heuristic Kalman Algorithm

R. Toscano * **P. Lyonnet** **

* *Universit de Lyon - Laboratoire de Tribologie et de Dynamique des Systmes CNRS UMR5513 ECL/ENISE, 58 rue Jean Parot 42023 Saint-Etienne cedex 2 (Tel: +33 477 43 84 84; e-mail: toscano@enise.fr).*
** *Universit de Lyon - Laboratoire de Tribologie et de Dynamique des Systmes CNRS UMR5513 ECL/ENISE, 58 rue Jean Parot 42023 Saint-Etienne cedex 2 (Tel: +33 477 43 84 84; e-mail: lyonnet@enise.fr).*

Abstract: This paper presents a simple but effective synthesis strategy for observers based faults detection in linear time-invariant (LTI) systems which are simultaneously affected by two classes of unknown inputs: Noises having fixed spectral densities and unknown finite energy disturbances. The problem of designing such an observer, also called a residual generator, is formulated as a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization problem. This is done to obtain an optimal residual generator, i.e. with minimal sensitivity to unknown inputs. Unfortunately, there is no known solution to this difficult optimization problem. Finding such a residual generator is known to be computationally intractable via the conventional techniques. This is mainly due to the non-convexity of the resulting optimization problem. To solve this kind of problem easily and directly, without using any complicated mathematical manipulations, we utilize the Heuristic Kalman Algorithm (HKA) for the resolution of the underlying constrained non-convex optimization problem. A numerical example is given to illustrate the advantage of the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization approach against techniques based on optimization of $\mathcal{H}_2$ or $\mathcal{H}_\infty$ criteria.

Keywords: Optimal residual generator; mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization problem; robust fault detection.

1. INTRODUCTION

It is a matter of fact that the high degree of automation in industrial process has enhanced the quality and efficiency of normal operation, but, in the same time, has also made systems more vulnerable to faults. As a consequence, dependability has become a central issue in all disciplines of systems engineering and software architecture. To assure a good level of dependability, the need for fault detection techniques have long been recognized; see Chen and Patton [1999] and Patton et al. [2000] for a good overview of works in this area. The fundamental purpose of a fault detection scheme is to "generate an alarm" when something is going wrong in the system. This can be done by using the concept of analytical redundancy. The principle rests on the comparison of the actual behaviour of the considered system to that expected via a mathematical model of this system. As a consequence, any inconsistency between the actual and expected behaviour can be interpreted as a fault. However, any real system is subject to unknown perturbations which also lead to inconsistency; therefore, any fault detection scheme needs to be sufficiently insensitive to these. Mathematically, the discrepancy between the actual and expected behaviour is expressed as residuals. Ideally, residuals are quantities that are nominally zero but they become non-zero if faults are present. In this paper, residuals will be generated with the help of a linear observer. Such an observer is then usually called a residuals generator. The basic principle of the linear observer-based residual generation consists in the estimation of the measured outputs of the system to be monitored. The resulting estimation error is then processed to decide whether or not a fault has occurred in the system. However, any real system is subject to perturbations which lead to nonzero residuals while there is no fault; therefore, it is highly desirable to minimize their effect on the residual generation. To this end, many approaches, such as $\mathcal{H}_\infty$ optimization, LMI, parity space and eigen-structure assignment techniques, have been applied to robust residual generator design with limited success (see for instance Liu at al. [2001], Sadrnia et al. [1996], Patton et al. [1991], Zhong et al. [2003]). The reason is that an efficient residual generator must satisfy contradictory objectives i.e. minimal sensitivity to the disturbances and maximal sensitivity to faults. Consequently, the design task must take into account these two conflicting requirements. Following this line, many approaches based $\mathcal{H}_2$ and/or, $\mathcal{H}_\infty$ criteria have been proposed in the literature (see for instance Liu and Zhou [2007], Henry and Zolghadri [2005], Liu et al. [2005], Rank and Niemann [1999], Wang et al. [2005]). However, most of them do not make distinction between the various sources of perturbations i.e.: noise, load disturbance and modelling errors. A notable exception is the work by Khosrowjerdi et al. [2005].

This paper deals with the fault detection of technical devices, in the presence of perturbations caused by noises, load disturbances and modelling errors. In what follows, these various perturbations are called unknown inputs. Roughly, these unknown inputs can be divided into two main classes (see Khosrowjerdi et al. [2005]): those having a fixed spectral densities (generally the noises) and those having a finite energy (usually the load disturbances and the modelling errors). As explained above, the minimization of the effects of these unknown inputs on the residual generation is of crucial importance to make a reliable fault detection. To this end, the problem of designing an observer-based residual generation, is formulated as a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization problem. This is done to obtain an optimal residual generator, i.e. with minimal sensitivity to unknown inputs. Unfortunately, there is no known solution to this difficult optimization problem. Finding such a residual generator is known to be computationally intractable via the conventional techniques. This is mainly due to the non-convexity of the resulting optimization problem. In Khosrowjerdi et al. [2005] a sub-optimal solution to this problem has been proposed. This was done by minimizing an upper bound of the original cost function.

In this paper, to solve the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem easily and directly, without using any complicated mathematical manipulations, we utilize the Heuristic Kalman Algorithm (HKA) for the resolution of the underlying constrained non-convex optimization problem. A numerical example is given to illustrate the advantage of the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization approach against existing techniques which are based on optimization of $\mathcal{H}_2$ or $\mathcal{H}_\infty$ criteria.

2. OPTIMAL RESIDUAL GENERATOR DESIGN BASED ON THE HEURISTIC KALMAN ALGORITHM (HKA)

In this section, a practical design procedure to determine the parameters of the residual generator is presented. To this end, we first formulate the problem of designing a robust residual generator as an optimization problem.

2.1 Formulation of the optimization problem

We assume that the system to be monitored can be described by the following state space model

$$\begin{cases} \dot{x}(t) = Ax(t) + B_u u(t) + B_v v(t) + B_w w(t) + B_f f(t) \\ y(t) = Cx(t) + D_u u(t) + D_v v(t) + D_w w(t) + D_f f(t) \\ x(0) = x_0 \end{cases} \tag{1}$$

where $x \in \mathbf{R}^{n_x}$ is the state vector, $u \in \mathbf{R}^{n_u}$ and $y \in \mathbf{R}^{n_y}$ are, respectively, the known input and output vectors. The unknown input $w \in \mathcal{R}^{n_w}$ represents the process/measurement noises, it is assumed to be of fixed spectral density. The unknown input $v \in \mathbf{R}^{n_v}$ is assumed to be a finite energy disturbance modelling errors caused by exogenous signals, linearization or parameter uncertainties. The unknown input $f \in \mathbf{R}^{n_f}$ is the fault vector; when $f = 0$, system (1) describes the fault-free system (i.e. the normal operating mode). The various constant matrices of (1) are assumed to be known and are of appropriate dimensions. It must be noticed that (1) is an augmented plant model which includes all the weighting functions reflecting the knowledge of w and v.

The objective is to develop a residual generator which generates, from the known input/output (i.e. $u(t)$ and $y(t)$), a set of residual signals $r(t)$ that are robust to unknown inputs (i.e. $v(t)$ and $w(t)$) and sensitive to the faults $f(t)$. In these conditions, we can conclude that a fault has occurred if some norm of $r(t)$ is larger than a prespecified threshold or if there are some changes in the statistical properties of the residual signals. This objective can be reached by using an observer-based residual generation. Consider then the following Luenbeger observer-based residual generation:

$$\begin{cases} \dot{z}(t) = Az(t) + B_u u(t) + L(y(t) - D_u u(t) - Cz(t)) \\ r(t) = y(t) - D_u u(t) - Cz(t) \\ z(0) = z_0 \end{cases} \tag{2}$$

where $z \in \mathbf{R}^{n_x}$ is the state vector of the observer and $L \in \mathbf{R}^{n_x \times n_y}$ is the matrix gains to be designed to ensure the stability of the observer as well as the robustness of the residuals to unknown inputs. Combining (1) and (2), we obtain:

$$\begin{cases} \dot{e}(t) = \tilde{A}e(t) + \tilde{B}_v v(t) + \tilde{B}_w w(t) + \tilde{B}_f f(t) \\ r(t) = Ce(t) + D_v v(t) + D_w w(t) + D_f f(t) \\ e(0) = e_0 \end{cases} \tag{3}$$

where $e = x - z$, $\tilde{A} = A - LC$, $\tilde{B}_v = B_v - LD_v$, $\tilde{B}_w = B_w - LD_w$ and $\tilde{B}_f = B_f - LD_f$. Note that the stability of the residual generator is guaranteed by ensuring that the matrix $\tilde{A}$ is Hurwitz. Taking the Laplace transform of (1), we obtain:

$$r(s) = G_e(s)e_0 + G_v(s)v(s) + G_w(s)w(s) + G_f(s)f(s) \tag{4}$$

where the transfert matrices $G_e(s)$, $G_v(s)$, $G_w(s)$ and $G_f(s)$ are defined as: $G_e(s) = C(sI - \tilde{A})^{-1}$, $G_v(s) = G_e(s)\tilde{B}_v + D_v$, $G_w(s) = G_e(s)\tilde{B}_w + D_w$ and $G_f(s) = G_e(s)\tilde{B}_f + D_f$.

We want the residual r insensitive to unknown inputs and initial conditions, and sensitive to faults. The stability of the observer will ensure the decay to zero of the effect of nonzero initial conditions e_0. Robustness as well as insensitivity to load disturbance can be achieved by satisfying $\|G_v(s, L)\|_\infty \leqslant \gamma$, with γ as small as possible. However, this kind of requirement can also lead to reduction of sensitivity to faults and to an increase of sensitivity to noise. Thus, in addition to $\|G_v(s, L)\|_\infty \leqslant \gamma$, we have to minimize the influence of noise and to maximize the effect of faults. This can be done by solving the following mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization problem:

$$\begin{aligned} &\text{Minimize} \quad J_1(L) = \frac{\|G_w(s, L) - D_w\|_2}{\|G_f(s, L)\|_\infty} \\ &\text{Subject to: } g_1(L) = \|G_v(s, L)\|_\infty - \gamma \leqslant 0 \\ &\qquad\qquad\quad g_2(L) = \arg\max_{\lambda_i(L)} \{\mathrm{Re}(\lambda_i(L)),\ \forall i\} - \lambda_{min} \leqslant 0 \\ &\qquad\qquad\quad L = [l_{ij}] = \begin{bmatrix} l_{11} & \cdots & l_{1n_y} \\ l_{21} & \cdots & l_{2n_y} \\ \vdots & \vdots & \vdots \\ l_{n_x 1} & \cdots & l_{n_x n_y} \end{bmatrix},\ \underline{l} \leqslant l_{i,j} \leqslant \bar{l} \end{aligned} \tag{5}$$

where $L = [l_{ij}]$ is the matrix of decision variables, $\underline{l}$ and $\bar{l}$ are the bounds of the hyperbox search domain. In the constraint $g_2(L)$, the quantity $\lambda_i(L)$ denotes the i^{th} pole of the observer. The parameter γ, is used to trade off between detection performance and noise sensitivity.

The constrained optimization problem (5), can be transformed into an unconstrained one, by introducing a new objective function which includes penalty functions:

$$J(q) = J_1(q) + \beta(\max(g_1(q), 0) + \max(g_2(q), 0)) \qquad (6)$$

where $q = \text{vect}(L)$ is the vector of decision variables defined as follows:

$$q = [l_{11}, \cdots, l_{1n_y}, l_{21}, \cdots, l_{2n_y}, \cdots, l_{n_x 1}, \cdots, l_{n_x n_y}] \qquad (7)$$

The setting of the weighting factor β is not very critical, it is only required to penalize more or less strongly the violations constraints. In all our experiment β has been set to 100. In these conditions we have to find the optimal vector of decision variables q_{opt}, defined as:

$$\begin{cases} q_{opt} = \arg\min_{q \in \mathcal{D}} J(q) \\ \mathcal{D} = \{q \in \mathbf{R}^{n_q} : \underline{l} \leqslant q_i \leqslant \bar{l}, \quad i = 1, \cdots, n_q\} \end{cases} \qquad (8)$$

where $n_q = n_x + n_y$. Unfortunately, the problem thus posed is known to be non-convex and thus computationally intractable. Therefore, due to the practical importance of the fault detection problem, it seems very useful to develop new design strategies for designing optimal robust residual generators. To this end, we present now a brief overview of HKA, which is capable of dealing with non-convex optimization problems. A more detailed study can be found in Toscano and Lyonnet [2008].

2.2 Resolution of the optimization problem via the HKA

The principle of HKA is depicted figure 1. The HKA includes a Gaussian random generator which produces, at each iteration, a collection of N vectors that are distributed about a given mean vector m_k with a given variance-covariance matrix Σ_k. This collection can be written as follows:

$$\mathbf{q}(k) = \left\{q_k^1, \; q_k^2, \; \cdots, \; q_k^N\right\} \qquad (9)$$

where q_k^i is the i^{th} vector generated at the iteration number k: $q_k^i = [q_{1,k}^i \cdots q_{n,k}^i]^T$, and $q_{l,k}^i$ is the l^{th} component of q_k^i $(l = 1, \cdots, n)$.

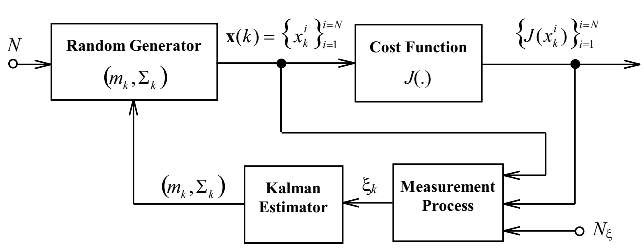

Fig. 1. Principle of the algorithm

This random generator is applied to the cost function J. Without loss of generality, we assume that the vectors are ordered by their increasing cost function i.e.:

$$J(q_k^1) < J(q_k^2) < \cdots < J(q_k^N) \qquad (10)$$

The principle of the algorithm is to modify the parameters of the gaussian generator so that its mean vector m_k, coincide with the optimum q_{opt}. More precisely, let N_ξ be the number of considered best samples, that is such that $J(q_k^{N_\xi}) < J(q_k^i)$ for all $i > N_\xi$. The problem is how to modify the parameters of the gaussian generator to achieve a reliable estimate of the optimum?

To solve this problem, a measurement process followed by a Kalman estimator is introduced. The measurement process consists in computing the average of the candidates that are the most representative of the optimum. For the iteration k, the measurement, denoted ξ_k, is then defined as follows:

$$\xi_k = \frac{1}{N_\xi} \sum_{i=1}^{N_\xi} q_k^i \qquad (11)$$

where N_ξ is the number of considered candidates. The Kalman estimator is used to update the parameters of the Gaussian generator in accordance with the informations drawn from the samples, i.e. the value of ξ_k and the variance vector associated to the best samples:

$$V_k = \frac{1}{N_\xi} \left[\sum_{i=1}^{N_\xi} (q_{1,k}^i - \xi_{1,k})^2, \cdots, \sum_{i=1}^{N_\xi} (q_{n,k}^i - \xi_{n,k})^2 \right]^T \qquad (12)$$

Based on the Kalman equations, the updating rules of the Gaussian generator are as follows (see Toscano and Lyonnet [2008], for a detailed derivation):

$$\begin{array}{l} m_{k+1} = m_k + L_k(\xi_k - m_k) \\ \Sigma_{k+1} = (I - a_k L_k)\Sigma_k \end{array} \qquad (13)$$

with :

$$\begin{cases} L_k = \Sigma_k(\Sigma_k + \text{diag}(V_k))^{-1}, \text{ and} \\ a_k = \dfrac{\alpha \min\left(1, \; \left(\frac{1}{n_q}\sum_{i=1}^{n_q} \sqrt{v_{i,k}}\right)^2\right)}{\min\left(1, \; \left(\frac{1}{n_q}\sum_{i=1}^{n_q} \sqrt{v_{i,k}}\right)^2\right) + \max\limits_{1 \leqslant i \leqslant n_q} (v_{i,k})} \end{cases} \qquad (14)$$

where $v_{i,k}$ represents the i^{th} component of the variance vector V_k defined in (12), and the scalar $\alpha \in (0, 1]$ is given by the designer. The flowchart of the HKA is given figure 2.

2.3 Initialization

The initial parameters of the Gaussian generator are selected to cover the entire search space. To this end, the following rule can be used:

$$m_0 = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_{n_q} \end{bmatrix}, \; \Sigma_0 = \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{n_q} \end{bmatrix} \qquad (15)$$

with:

$$\begin{cases} \mu_i = \dfrac{\bar{l} + \underline{l}}{2} \\ \sigma_i = \dfrac{\bar{l} - \underline{l}}{6} \end{cases} \qquad (16)$$

where $\bar{l}$ (respectively $\underline{l}$) is the upper bound (respectively lower bound) of the hyperbox search domain. With this rule, 99% of the samples are generated in the intervals $\mu_i \pm 3\sigma_i$, $i = 1, \ldots, n_q$.

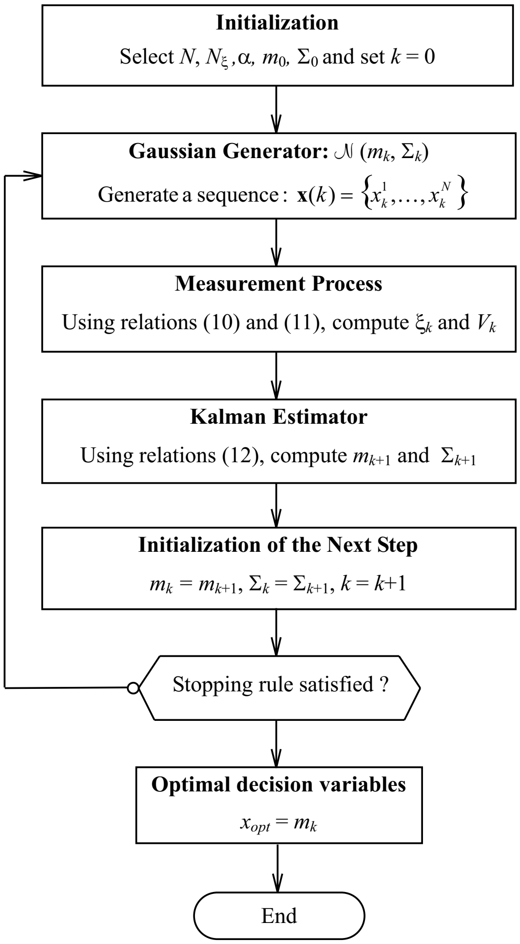

Fig. 2. Flow chart of the HKA.

3. NUMERICAL EXPERIMENTS

In this section, to illustrate the usefulness of the proposed optimization approach, we consider the problem of fault detection in a four-tank process. The state-space process model is given by

$$
\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} -0.0159 & 0 & 0.0419 & 0 \\ 0 & -0.0111 & 0 & 0.0333 \\ 0 & 0 & -0.0419 & 0 \\ 0 & 0 & 0 & -0.0333 \end{bmatrix} x(t) \\
&+ \begin{bmatrix} 0.0833 & 0 \\ 0 & 0.0718 \\ 0 & 0.0479 \\ 0.0312 & 0 \end{bmatrix} (u(t) + f(t)) \\
&+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.0357 & 0 \\ 0 & -0.0313 \end{bmatrix} v(t) \\
y(t) &= \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix} x(t) + w(t)
\end{aligned}
\tag{17}
$$

where the state vector $x = [x_1 \ x_2 \ x_3 \ x_4]^T$ represents the level of water in the tanks, the control input $u = [u_1 \ u_2]^T$ is the voltage applied to the pumps, $f = [f_1 \ f_2]^T$ is the fault vector associated to the pumps, $v = [v_1 \ v_2]^T$ is the disturbance vector and $w = [w_1 \ w_2]^T$ is the measurement noise vector. The objective is to detect the actuator fault f in the presence of a disturbance v and the measurement noise w. To evaluate the performance of the residual generator, a fault f_1 (see Fig. 3) and a disturbance v_1 (see Fig. 4) are applied.

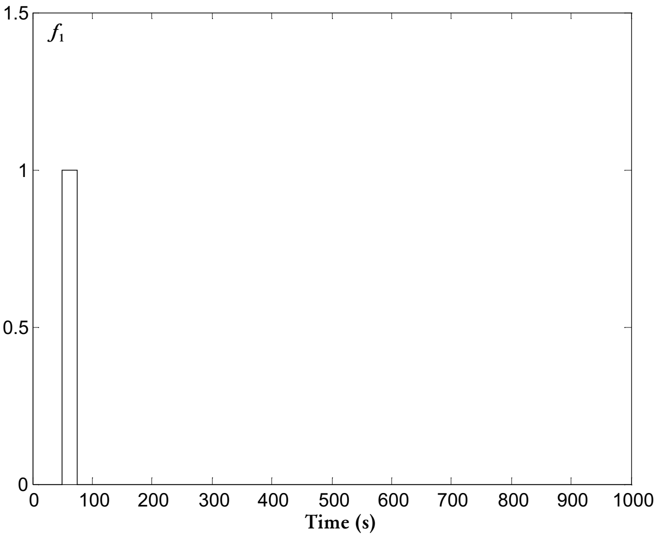

Fig. 3. Pump fault f_1.

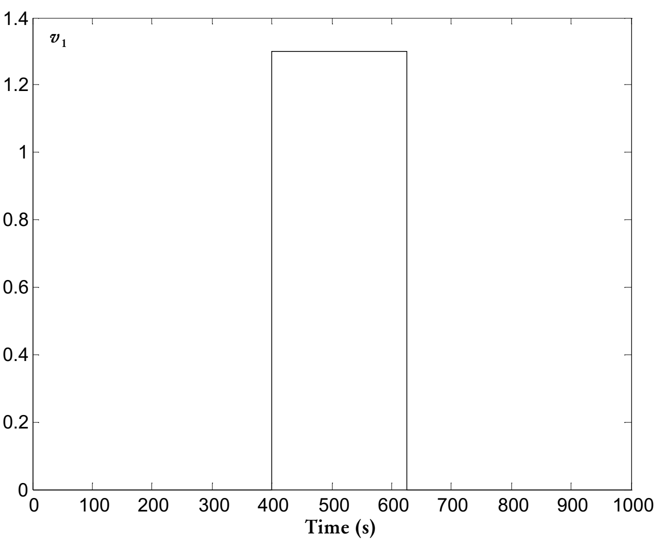

Fig. 4. Disturbance v_1.

The synthesis of the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ residual generator was done by solving the optimisation problem (5) via HKA. The following parameters have been used: $N = 50$, $N_\xi = 5$, $\alpha = 0.4$, $\underline{l} = -1$, $\bar{l} = 1$, $\lambda_{min} = -0.01$ and $\gamma = 0.08$. Fig. 5 shows the simulation result obtained with the resulting residual generator. This figure describes the evolution of the absolute value of the residual $r_1(t)$. We can see that the effect of the disturbance $v_1(t)$ on the residual $r_1(t)$ is strongly attenuated and the effect of the fault is significantly bigger than that of $v_1(t)$. Therefore, this fault can be easily detected by using an appropriate threshold. The ratio between the maximum value of the effect of the fault to the maximum value of the effect of the disturbance is 2.6. This ratio is only of 1.8 by using the approach proposed by Khosrowjerdi et al. [2005]. This clearly shows the better performance of the proposed approach. Indeed, HKA allows to solve directly the optimisation problem (5) without using any upper bound nor transforming the non-convex problem into a convex one, as it is the case in Khosrowjerdi et al. [2005].

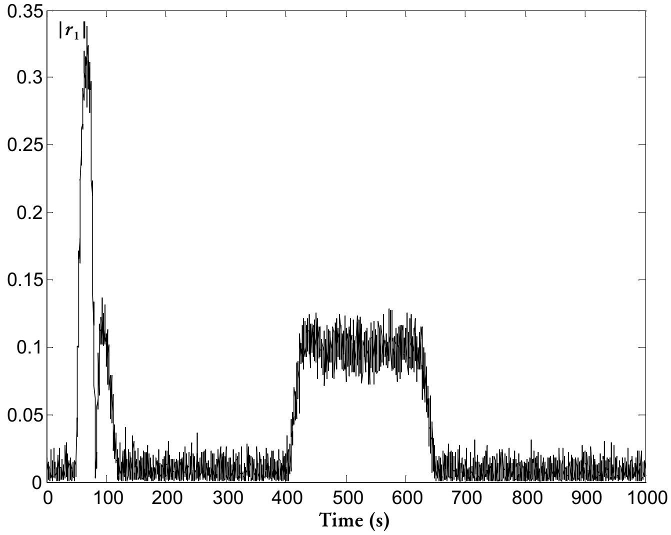

Fig. 5. Evolution of $|r_1(t)|$, mixed $\mathcal{H}_2/\mathcal{H}_\infty$, $\gamma = 0.08$.

For comparison, Fig. 6 shows the result obtained when the residual generator is designed by just solving the $\mathcal{H}_2$ optimisation problem

$$\begin{cases} L_{opt} = \arg\min_{L} \|[G_v(s,L)\ G_w(s,L)]\|_2 \\ g(L) = \arg\max_{\lambda_i(L)} \{\mathrm{Re}(\lambda_i(L)),\ \forall i\} - \lambda_{min} \leqslant 0 \end{cases} \tag{18}$$

Similarely, Fig. 7 shows the result obtained by just solving the $\mathcal{H}_\infty$ optimisation problem

$$\begin{cases} L_{opt} = \arg\min_{L} \|[G_v(s,L)\ G_w(s,L)]\|_\infty \\ g(L) = \arg\max_{\lambda_i(L)} \{\mathrm{Re}(\lambda_i(L)),\ \forall i\} - \lambda_{min} \leqslant 0 \end{cases} \tag{19}$$

As we can see, the corresponding residual generators cannot be used to detect the fault $f_1(t)$. This confirm the usefulness of a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis.

4. CONCLUSION

In this paper, a straightforward design method for robust residual generator satisfying mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance criteria was developed. This sort of estimation problem usually results in a non-convex constrained optimization problem which is known to be very difficult to deal with. This is why, to solve in a direct way this kind of problem, we have proposed to use the Heuristic Kalman Algorithm. Indeed, HKA runs without any conservative assumption usually required in the conventional methods, in addition it allows to determine residual generator gains by solving the constrained optimization problem in a direct way without requiring any complicated mathematical manipulations. Simulation studies have demonstrated the validity of the proposed approach, in particular a comparisons with the work presented in Khosrowjerdi et al. [2005], has shown that a direct resolution of the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization problem via HKA leads to better results, notably concerning the detectability of a fault.

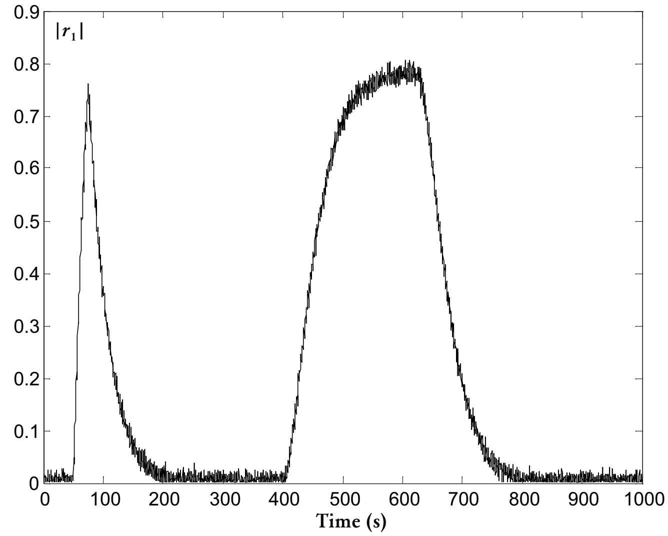

Fig. 6. Evolution of $|r_1(t)|$, $\mathcal{H}_2$ synthesis.

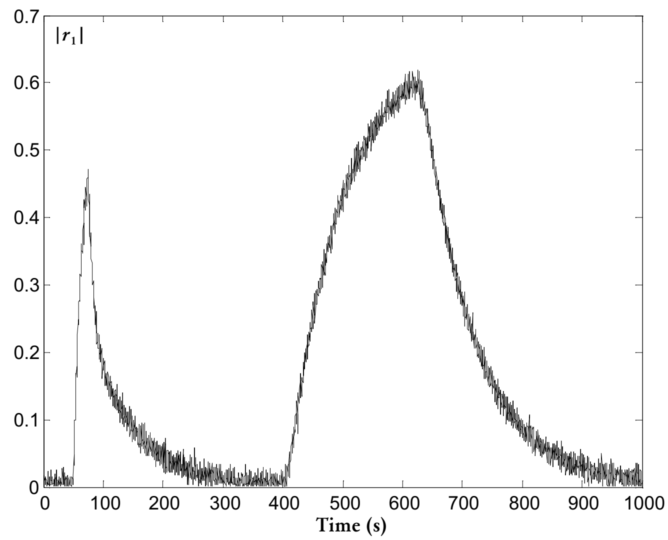

Fig. 7. Evolution of $|r_1(t)|$, $\mathcal{H}_\infty$ synthesis.

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