A Kalman optimization approach for solving some industrial electronics problems

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***Abstract*—This paper is concerned with solving non-convex optimization problems arising in various engineering sciences. In particular, we focus on the design of a robust flux estimator of induction machines and the optimal design of on-chip spiral inductors. To solve these problems, a recently developed optimization method, called the heuristic Kalman algorithm (HKA), is employed. The principle of HKA is to explicitly consider the optimization problem as a measurement process designed to give an estimate of the optimum. A specific procedure, based on the Kalman estimator, was developed to improve the quality of the estimate obtained through the measurement process. The main advantage of HKA, compared to other stochastic optimization methods, lies in the small number of parameters that need to be set by the user. Based on HKA a simple but effective design strategy for robust flux estimator and on-chip spiral inductors is developed. Numerical studies are conducted to demonstrate the validity of the proposed design procedure.**

I. Introduction

THE problem of estimating the true value of a given variable which is subject to some stochastic disturbances is a major concern in many disciplines such as electrical engineering, mechanical engineering, chemical engineering, robotics, economics, to cite only a few. It is well known that such problems can be solved optimally, in the sense of a minimum variance, using the Kalman filter.

A less known aspect of the Kalman filter is that it can be used to solve non-convex optimization problems. This is a very interesting fact because a lot of engineering problems are formulated as non-convex optimization problems and thus our ability in solving them is of crucial importance. However, the non-convex problems are known to be difficult to solve, it is indeed now well recognized that the great watershed in optimization is not between linearity and nonlinearity, but convexity and non-convexity.

Very efficient algorithms for solving convex problems exist [3], whereas the problem of non-convex optimization remains largely open despite an enormous amount of effort devoted to its resolution.

One of the main objectives of this paper is to introduce a new optimization method, called the heuristic Kalman algorithm (HKA) able to deal with non-convex problems. This approach falls in the category of the so-called "population-based stochastic optimization techniques". However, its search heuristic is entirely different from other known stochastic algorithms [16], [5], [11]. Indeed, HKA explicitly considers the optimization problem as a measurement process designed to give an estimate of the optimum. A specific procedure, based on the Kalman estimator, was developed to improve the quality of the estimate obtained through the measurement process. The main advantage of HKA compared to other metaheuristics, lies in the small number of parameters that need to be set by the user (only three). This property makes the algorithm easy to use for non-specialists.

On the other hand, this paper is concerned with the design of a robust flux estimator of induction machines and the optimal design of on-chip spiral inductors. These two domains of application have been chosen due to their practical importance in industrial electronics.

The Induction motor is widely used in industry mainly because of its simple and robust structure which results in a very reliable operation. However, the control of this kind of actuator is very difficult due to its highly nonlinear dynamic. Field-oriented control has proved to be an efficient approach to the control of induction machine and continues to be an active research area [6], [10]. However, most of the practical implementations of this technique require the knowledge of the rotor flux which is not available in industrial machines. Consequently, rotor flux estimation from the stator variables (voltage and current) and rotor speed is an important issue, [21], [9], [15], [7], [19]. Despite many contributions, the problem of designing a flux estimator remains a challenging task. This is mainly due to the fact that parameter uncertainties, such as the variation of the rotor resistance, affects significantly the dynamic of the motor and thus increases the estimation error. It is then necessary to take into account the parameter uncertainties to make the flux estimator less sensitive to uncertainties. To this end, a robust observer for flux estimation is proposed. The matrix gain of the observer is designed to ensure a minimal sensitivity to parameter uncertainties and noise measurement while ensuring the robust stability of the estimator. It is shown that this results in a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization problem including a structural constraint on the matrix gain.

Thus formulated, this problem is extremely difficult to solve in the framework of LMI (Linear Matrix Inequalities [2]) because of the structural constraint on the matrix gain. In addition, a major drawback with LMI approaches is the use of Lyapunov variables, whose number grows quadratically with

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the system size, whereas we are looking for the structured matrix gain observer which contains a comparatively very small number of unknowns. It is then necessary to introduce new techniques capable of dealing with this kind of optimization problems without introducing extra unknown variables and without using too many user defined parameters. As we will see, this is precisely what the HKA method can do.

The second domain of application considered in this paper is the design of on-chip spiral inductors [20], [1], [8]. This kind of component is an essential part of any radio frequency integrated circuit such as voltage controlled oscillators, low-noise amplifiers etc. Consequently, the optimal design of this kind of component is of great practical importance. Typically, we have to determine the layout parameters to obtain the desired value of the inductance. But this is not sufficient, because at high frequencies (i.e. in the Ghz range), some complicated losses mechanisms must be taken into account to make a realistic design. Theoretically, an exact design can be done by solving Maxwells equations and practically a good numerical solution may be obtained by using available field solvers like for instance ASITIC [14]. However, field solvers are computationally intensive and require long run times, and so are more appropriate for design verification than the design stage of an inductor. The approach adopted in this work is to use a simplified model of the on-chip spiral inductor that can predict its behaviour in a broad range of frequencies. Based on this model, we can design, quickly, high performance spiral inductors (i.e. with minimum losses) through the use of optimization techniques. However the resulting optimization problem is non-convex and thus extremely difficult to solve via conventional techniques. Using the field solver ASITIC as a verification tool, it is shown that the Kalman optimization method is a good alternative for solving this kind of problem.

The remaining part of this paper is organized as follows. In section 2, the heuristic Kalman algorithm (HKA), is presented. In particular, we describe the main components of the HKA: Gaussian generator, measurement process and Kalman estimator. The updating rules of the HKA are then introduced and the problem of initialization and parameters setting is discussed. Section 3 presents the robust flux estimator design based on the heuristic Kalman algorithm (HKA). Section 4 is devoted to the optimal design on-chip spiral inductors via HKA. Finally, section 5 concludes this paper.

II. The heuristic Kalman algorithm (HKA)

Optimization is the way of obtaining the best possible outcome given the degrees of freedom and the constraints. More formally, an optimization problem has the following form:

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leqslant 0, \quad i = 1, \cdots, N_c \\ & x \in \mathcal{D} = \{x \in \mathbf{R}^{n_x} : \underline{x} \preceq x \preceq \bar{x}\} \end{array} \tag{1}$$

where $f_0 : \mathbf{R}^{n_x} \rightarrow \mathbf{R}$ is the objective function (or cost function) i.e. the function that we want to minimize[1], $f_i : \mathbf{R}^{n_x} \rightarrow \mathbf{R}$, $i = 1, \cdots, N_c$ are the constraint functions, and the vector $x = (x_1, \cdots, x_{n_x})$ is the optimization variable also called decision variable or design variable. The set $\mathcal{D}$ is such we call the search domain[2] i.e. the set under which the minimization is performed. The vectors $\underline{x} = (\underline{x}_1, \cdots, \underline{x}_{n_x})$ and $\bar{x} = (\bar{x}_1, \cdots, \bar{x}_{n_x})$ are the bounds of the search domain and the symbol $\preceq$ means a componentwise inequality. A vector $x_f \in \mathcal{D}$ is said feasible if it satisfies the N_c constraints f_i; the set of feasible vector is called the feasible domain. We denote by x_{opt} a solution of the problem (1), i.e. a vector which ensures the smallest objective value among all vectors that satisfy the constraints.

The functional constraints f_i can be handled by introducing a new objective function including penalty functions:

$$J(x) = f_0(x) + \sum_{i=1}^{N_c} w_i \max(f_i(x), 0) \tag{2}$$

Where N_c is the number of constraints and the w_i's are weighting factors. There exists a vast literature dealing with the problem of the updating rule of the weighting factor (see for instance [4]). However, in most practical applications, the choice of constant weighting factors leads to a satisfying solution (possibly sub-optimal). In this case, the setting of the w_i's must be done to penalize more or less strongly the violation constraints. Note that if x satisfies the constraints then $J(x) = f_0(x)$. In these conditions solving problem (1) is the same as solving the following optimization problem:

$$\begin{array}{ll} \text{minimize} & J(x) \\ \text{subject to} & x \in \mathcal{D} = \{x \in \mathbf{R}^{n_x} : \underline{x} \preceq x \preceq \bar{x}\} \end{array} \tag{3}$$

Thus posed, the objective is then to find the optimum x_{opt} i.e. the n_x-dimensional decision vector which minimizes the cost function J. Unfortunately, there are several obstacles for solving this kind of problem. The main obstacle is that most of the optimization problems are NP-hard. Therefore the known theoretical methods cannot be applied except possibly for some small size problems. Other difficulties are that the cost function may be not differentiable and/or multimodal. Therefore the set of methods requiring the derivatives of the cost function cannot be used. Another obstacle is when the cost function cannot be expressed in an analytic form, in this case, the cost function can be only evaluated through simulations.

In these situations, heuristic approaches seem to be the only way for solving optimization problems. By heuristic approach, we mean a computational method employing experimentations, evaluations and trial-and-errors procedures to obtain an approximate solution for computationally difficult problems. The HKA described in the next section, was built with this in mind.

A. Principle of the algorithm

The principle of the algorithm is depicted in figure 1. The proposed procedure is iterative, and we denote by k, the k^{th} iteration of the algorithm. The HKA includes a Gaussian random generator which produces, at each iteration, a collection

[1]Note that any maximization problem can be converted into a minimization problem. Indeed maximizing $f(x)$ is the same as minimizing $-f(x)$.

[2]The set $\mathcal{D}$ is a hyperbox and thus is also called the hyperbox search domain.

of N vectors that are normally distributed according to the mean value m_k and the standard deviation vector S_k of the Gaussian generator. This collection can be written as follows:

$$\mathbf{x}(k) = \left\{x_k^1,\ x_k^2,\ \cdots,\ x_k^N\right\} \tag{4}$$

where x_k^i is the i^{th} vector generated at the iteration number k: $x_k^i = [x_{1,k}^i \cdots x_{n,k}^i]^T$, and $x_{l,k}^i$ is the l^{th} component of x_k^i $(l = 1, \cdots, n)$.

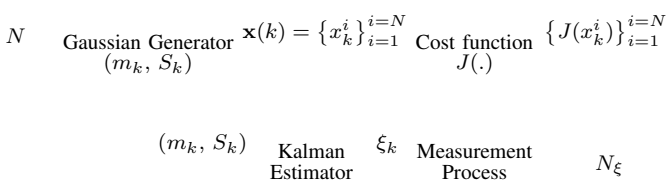

Fig. 1. Principle of the algorithm

This random generator is applied to the cost function J. Without loss of generality, we assume that the vectors are ordered by their increasing cost function i.e.:

$$J(x_k^1) < J(x_k^2) < \cdots < J(x_k^N) \tag{5}$$

The principle of the algorithm is to modify the mean vector and the standard deviation vector of the random generator in order to decrease the cost function value. This procedure is then repeated until no more improvement can be found. More precisely, let N_ξ be the number of considered best samples, that is such that $J(x_k^{N_\xi}) < J(x_k^i)$ for all $i > N_\xi$. Note that the best samples are those of sequence (4) which have the smallest cost function. The objective is then to generate, from the best samples, a new random distribution in order to improve the current solution.

To this end, a measurement procedure followed by an optimal estimator of the parameters of the random generator is introduced. The measurement process consists in computing the average of the best candidate solutions. For the k^{th} iteration, the measurement, denoted ξ_k, is defined as follows:

$$\xi_k = \frac{1}{N_\xi}\sum_{i=1}^{N_\xi} x_k^i \tag{6}$$

where N_ξ is the number of considered candidates. It can be considered that this measure gives a perturbed knowledge about the optimum, i.e.

$$\xi_k = x_{opt} + v_k \tag{7}$$

where v_k is an unknown zero-mean perturbation acting on the measurement process.

Note that v_k is the random error vector between the measure ξ_k and the unknown optimum x_{opt}. In other words, v_k is a kind of measure of our ignorance about x_{opt}. Of course, this uncertainty cannot be measured but only estimated by taking into account all available knowledge. In our case, the uncertainty of the measure is closely related to the dispersion of the best samples x_k^i, $i = 1, \cdots, N_\xi$. Our ignorance about the optimum can thus be taken into account by using the variance vector associated to these best samples:

$$V_k = \frac{1}{N_\xi}\left[\sum_{i=1}^{N_\xi}(x_{1,k}^i - \xi_{1,k})^2, \cdots, \sum_{i=1}^{N_\xi}(x_{n,k}^i - \xi_{n,k})^2\right]^T \tag{8}$$

In these conditions, the Kalman filter can then be used to make an estimate, so-called "a posteriori", of the optimum, i.e. taking into account the measure as well as the confidence we place in it. As seen, this confidence can be quantified by the variance vector (8).

Roughly speaking, a Kalman filter is an optimal recursive data processing algorithm [12]. Optimality must be understood as the best estimate that can be made based on the model used for the measurement process as well as the data used to compute this estimate.

B. Equations of the Kalman estimator

The objective is to design an optimal estimator which combines a prior estimation of x_{opt} and the measurement ξ_k, so that the resulting posterior estimate is optimal in a sense which will be defined below. In the Kalman framework, this kind of estimator takes the following form:

$$\hat{x}_k^+ = L_k'\hat{x}_k^- + L_k\xi_k \tag{9}$$

where $\hat{x}_k^-$ represents the prior estimation i.e. before the measurement, $\hat{x}_k^+$ is the posterior estimation i.e. after the measurement, L_k' and L_k are unknown matrices which have to be determined to ensure an optimal estimation. Here optimality is reached when the expectation of the posterior estimation error is zero and its variance is minimal. This can be expressed as follows:

$$(L_k',\ L_k) = \arg\min_{L_k',\ L_k} \mathbf{E}[\tilde{x}_k^{+T}\tilde{x}_k^+], \quad \mathbf{E}[\tilde{x}_k^+] = 0 \tag{10}$$

where $\mathbf{E}$ is the expectation operator and $\tilde{x}_k^+$ represents the posterior estimation error at iteration k. We define the posterior estimation error $\tilde{x}_k^+$ and its variance-covariance matrix P_k^+ as:

$$\tilde{x}_k^+ = x_{opt} - \hat{x}_k^+, \quad P_k^+ = \mathbf{E}[\tilde{x}_k^+\tilde{x}_k^{+T}] \tag{11}$$

In the same way, we define the prior estimation error $\tilde{x}_k^-$ and its variance-covariance matrix P_k^- as:

$$\tilde{x}_k^- = x_{opt} - \hat{x}_k^-, \quad P_k^- = \mathbf{E}[\tilde{x}_k^-\tilde{x}_k^{-T}] \tag{12}$$

Under the assumption that $\mathbf{E}[\tilde{x}_k^-] = 0$, it can be easily established that the satisfaction of the condition $\mathbf{E}[\tilde{x}_k^+] = 0$ requires

$$L_k' = I - L_k \tag{13}$$

where I is the identity matrix. Then, putting this expression into equation (9) gives:

$$\hat{x}_k^+ = \hat{x}_k^- + L_k(\xi_k - \hat{x}_k^-) \tag{14}$$

The objective is now to determine L_k in such a way that the variance of the posterior estimation error is minimized. Noting that: $\text{trace}(P_k^+) = \mathbf{E}[\tilde{x}_k^{+T}\tilde{x}_k^+]$, the minimization of the variance of $\tilde{x}_k^+$ is accomplished by minimizing the trace of

P_k^+ with respect to L_k. A standard calculus, similar to the one used for the derivation of the Kalman filter (see [12]), yields:

$$L_k = P_k^-(P_k^- + \text{diag}(V_k))^{-1}, \quad P_k^+ = (I - L_k)P_k^- \qquad (15)$$

where $\text{diag}(V_k)$ is a diagonal matrix having in its diagonal the variance vector V_k.

C. Updating rule of the Gaussian generator

In the HKA, $(\hat{x}_k^-, \text{vec}^d(P_k^-))$ represents the mean value and the variance vector[3] of the Gaussian generator at iteration k i.e. $m_k = \hat{x}_k^-$ and $S_k = (\text{vec}^d(P_k^-))^{1/2}$ (recall that S_k represents the standard deviation vector of the Gaussian generator). According to the Kalman equations (14), (15), the updating rule of the Gaussian generator are given by $m_{k+1} = \hat{x}_k^+$ and $S_{k+1} = (\text{vec}^d(P_k^+))^{1/2}$. However, the expression for computing P_k^+ (see 15) generally leads to a decrease in the variance of the Gaussian distribution that is too fast, which results in a premature convergence of the algorithm. This difficulty can be tackled by introducing a slowdown factor that is adjusted according to the dispersion of the best candidates considered for the improvement of the current solution. This can be done as follows:

$$S_{k+1} = S_k + a_k(W_k - S_k) \qquad (16)$$

with:

$$\begin{cases} a_k = \dfrac{\alpha \min\left(1, \left(\frac{1}{n_x}\sum_{i=1}^{n_x}\sqrt{v_{i,k}}\right)^2\right)}{\min\left(1, \left(\frac{1}{n_x}\sum_{i=1}^{n_x}\sqrt{v_{i,k}}\right)^2\right) + \max_{1\leqslant i\leqslant n_x}(w_{i,k})} \\ S_k = \left(\text{vec}^{\text{d}}(P_k^-)\right)^{1/2},\ W_k = \left(\text{vec}^{\text{d}}(P_k^+)\right)^{1/2} \end{cases}, \qquad (17)$$

where a_k is the slowdown factor, $\alpha \in (0,\ 1]$ the slowdown coefficient, and $v_{i,k}$ represents the i^{th} component of the variance vector $\text{vec}^{\text{d}}(V_k)$ defined in (8), $w_{i,k}$ is the i^{th} component of the vector W_k, and $\text{vec}^{\text{d}}(.)$ is the diagonal vector of the matrix given in argument.

All the matrices used in our formulation (i.e. P_k^+, P_k^-, L_k) are diagonal. Consequently, to save computation time, we must use a vectorial form for computing the various quantities of interest. According to (14), (15) (16) and (17), the updating rule of the Gaussian generator can be rewritten in a vectorial form as follows:

$$\begin{array}{l} m_{k+1} = m_k + L_k \circledast (\xi_k - m_k),\ S_{k+1} = s_k + a_k(W_k - S_k) \\ L_k = S_k^2 // (S_k^2 + V_k),\ W_k = (S_k^2 - L_k \circledast S_k^2)^{1/2} \end{array} \qquad (18)$$

where the symbol $\circledast$ stands for a componentwise product and $//$ represents a componentwise divide. The heuristic Kalman algorithm can then be summarized as follows.

1) Initialization. *Choose N, N_ξ and α. Set $m_k := m_0$ and $S_k := S_0$.*
2) Gaussian generator (m_k, S_k). *Generate a sequence of N vectors $\mathbf{x}(k) = \left\{x_k^1,\ x_k^2,\ \cdots,\ x_k^N\right\}$ according to a Gaussian distribution parametrized by m_k and S_k.*
3) Measurement process. *Using relations (6) and (8), compute ξ_k and V_k.*
4) Updating rule of the Gaussian generator. *Using relation (18), compute m_{k+1} and S_{k+1}.*
5) Initialization of the next step. *Set $m_k := m_{k+1}$ and $S_k := S_{k+1}$.*
6) Termination test. *If the Stopping rule is not satisfied, go to step 2, otherwise stop.*

A detailed discussion on the convergence property of this algorithm can be found in [17], [18].

[3]The notation $\text{vec}^d(.)$ represents the diagonal vector of the matrix passed in argument.

TABLE I
EFFECT OF THE HKA PARAMETERS (↗ : INCREASE, ↘ : DECREASE).

Parameter	N ↗	N_ξ ↗	α ↗
Number of function evaluations	↗	↗	↘
Typical values	20-150	N/10	0.4-0.9

D. Initialization and parameter settings

The initial parameters of the Gaussian generator are selected to cover the entire search space. To this end, the following rule can be used:

$$m_0 = [\mu_1, \cdots, \mu_{n_x}]^T,\ S_0 = [\sigma_1, \cdots, \sigma_{n_x}]^T \qquad (19)$$

with:

$$\mu_i = \frac{\bar{x}_i + \underline{x}_i}{2},\ \sigma_i = \frac{\bar{x}_i - \underline{x}_i}{6}, \quad i = 1, \ldots, n_q, \qquad (20)$$

where $\bar{x}_i$ (respectively, $\underline{x}_i$) is the i^{th} upper bound (respectively, lower bound) of the hyperbox search domain. With this rule, 99% of the samples are generated in the interval $\mu_i \pm 3\sigma_i$, $i = 1, \ldots, n_x$. The three following parameters must be set: the number of points N, the number of best candidates N_ξ, and the coefficient α. To facilitate this task, TAB. I summarizes the influence of these parameters on the number of function evaluations and thus on the CPU time. This table gives also some typical values of the user defined parameters.

III. ROBUST OBSERVER FOR ROTOR FLUX ESTIMATION OF AN INDUCTION MACHINE

The design of a robust flux estimator requires a model of the induction machine that takes into account the perturbations due to parametric uncertainties as well as the noise measurement. This uncertain model can then be used to develop a robustly stable flux observer with minimal sensitivity to disturbances. In what follows, these various aspects will be considered with some details.

A. Induction motor model including parametric disturbances and noise measurement

Under the assumptions of linearity and symmetry of electric and magnetic circuits and neglecting iron losses, the dynamic model of a squirrel-cage induction motor in the fixed stator reference frame can be written as follows:

$$\begin{array}{l} \dot{x}(t) = \tilde{A}(\omega)x(t) + Bu(t), \\ \tilde{A}(\omega) = \begin{bmatrix} \tilde{a}_1 & 0 & \tilde{a}_2 & a_3\omega(t) \\ 0 & \tilde{a}_1 & -a_3\omega(t) & \tilde{a}_2 \\ \tilde{a}_4 & 0 & \tilde{a}_5 & -n_p\omega \\ 0 & \tilde{a}_4 & n_p\omega & \tilde{a}_5 \end{bmatrix},\ B = \begin{bmatrix} b & 0 \\ 0 & b \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array} \qquad (21)$$

where ω is the rotor speed, $x = [i_\alpha,\, i_\beta, \varphi_\alpha,\, \varphi_\beta]^T$ is the state vector, i_α, i_β are the stator currents, φ_α, φ_β are the rotor fluxes and $u = [u_\alpha,\, u_\beta]^T$ represents the stator voltage. The parameters of the induction machine are: the uncertain but bounded stator resistance $\tilde{R}_s \in [\underline{R}_s,\, \bar{R}_s]$, the uncertain but bounded rotor resistance $\tilde{R}_r \in [\underline{R}_r,\, \bar{R}_r]$, the stator inductance L_s, the rotor inductance L_r, the mutual inductance L_{sr} and the number of pole pairs n_p. The bounds of variation of the stator and rotor resistances (i.e. $\underline{R}_s$, $\bar{R}_s$, $\underline{R}_r$ and $\bar{R}_r$) are assumed to be known. The entries of the matrices $\tilde{A}(\omega)$ and B are defined as: $\tilde{a}_1 = \tilde{a}_{11} + \tilde{a}_{12}$, $\tilde{a}_{11} = a_{11}\tilde{R}_s$, $a_{11} = -\frac{1}{\sigma L_s}$, $\sigma = 1 - \frac{L_{sr}^2}{L_s L_r}$, $\tilde{a}_{12} = a_{12}\tilde{R}_r$, $a_{12} = -\frac{L_{sr}^2}{\sigma L_s L_r^2}$, $\tilde{a}_2 = a_2\tilde{R}_r$, $a_2 = \frac{L_{sr}}{\sigma L_s L_r^2}$, $a_3 = n_p\frac{L_{sr}}{\sigma L_s L_r}$, $\tilde{a}_4 = a_4\tilde{R}_r$, $a_4 = \frac{L_{sr}}{L_r}$, $\tilde{a}_5 = a_5\tilde{R}_r$, $a_5 = -\frac{1}{L_r}$, $b = \frac{1}{\sigma L_s}$.

It is interesting to note that the evolution matrix $\tilde{A}(\omega)$ can be rewritten as $\tilde{A}(\omega) = \tilde{R}_r A_r + \tilde{R}_s A_s + \omega A_\omega$, where A_r, A_s and A_ω are constant matrices defined as follows

$$A_r = \begin{bmatrix} a_{12} & 0 & a_2 & 0 \\ 0 & a_{12} & 0 & a_2 \\ a_4 & 0 & a_5 & 0 \\ 0 & a_4 & 0 & a_5 \end{bmatrix},\ A_s = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_\omega = \begin{bmatrix} 0 & 0 & 0 & a_3 \\ 0 & 0 & -a_3 & 0 \\ 0 & 0 & 0 & -n_p \\ 0 & 0 & n_p & 0 \end{bmatrix} \tag{22}$$

System (21) can then be expressed as:

$$\begin{aligned} &\dot{x}(t) = A(\omega)x(t) + Bu(t) + B_w w(t), \\ &A(\omega) = R_r A_r + R_s A_s + \omega A_\omega,\ \ B_w = [R_r' A_r \ \ R_s' A_s], \\ &R_r = \tfrac{\bar{R}_r + \underline{R}_r}{2},\ R_r' = \tfrac{\bar{R}_r - \underline{R}_r}{2},\ R_s = \tfrac{\bar{R}_s + \underline{R}_s}{2},\ R_s' = \tfrac{\bar{R}_s - \underline{R}_s}{2} \end{aligned} \tag{23}$$

where w is the unknown vector of disturbances due to the parametric uncertainties. The measured variables are the stator currents and the rotor speed. The output equation is then given by:

$$\begin{aligned} &y_i(t) = C_i x(t) + D_i v_i(t), \quad y_\omega(t) = \omega(t) + v_\omega(t) \\ &C_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ D_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \tag{24}$$

where y_i is the vector of measured currents, y_ω is the measured rotor speed, v_i and v_ω are the noises measurement. Finally, the complete model of the induction motor that takes into account the perturbations due to parametric uncertainties and the noise measurement is given by equations (22), (23) and (24).

B. Robustly stable flux observer

Any observer utilizes a real-time simulation of the system model corrected by the estimation error. This principle leads to the following flux observer:

$$\begin{aligned} &\dot{\hat{x}}(t) = A(y_\omega)\hat{x}(t) + Bu(t) + K(y_i(t) - \hat{y}_i(t)) \\ &\hat{y}_i(t) = C_i\hat{x}(t), \ \ A(y_\omega) = R_r A_r + R_s A_s + y_\omega A_\omega \end{aligned} \tag{25}$$

The problem is to determine the matrix gain to ensure the robust stability of this time varying observer (the evolution matrix depends upon the rotor speed measurement) while ensuring a minimal sensitivity to parametric uncertainties and noise measurement. The error dynamic is given by:

$$\begin{aligned} \dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= (A(\omega) - KC_i)e(t) + B_w w(t) - D_v v(t) \end{aligned} \tag{26}$$

where $D_v = [KD_i \ \ A_\omega]$, and v is the noise measurement. Under the assumption that $\omega \in [\underline{\omega},\, \bar{\omega}]$, with $\bar{\omega} > 0$ and $\underline{\omega} = -\bar{\omega}$, it can be shown that if the matrix gain satisfies the following structural constraint:

$$K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ k_2 & k_1 & k_4 & k_3 \end{bmatrix}^T \tag{27}$$

and is such that $A(\underline{\omega}) - KC_i$ is Hurwitz, then the time varying flux observer (25) is asymptotically stable i.e. when $w(t) = 0$, $v(t) = 0$ and $\omega(t) \in [\underline{\omega},\, \bar{\omega}]$, we have $\lim_{t\to\infty} e(t) = 0$. In addition to this stability condition it is necessary to ensure that the estimation error remains small for non-zero disturbances. In what follows, it is shown that this requirement can be formulated as an optimization problem.

C. Formulation of the optimization problem for a minimal sensitivity to disturbances

Taking the Laplace transform of (26) for a given constant value of ω, gives:

$$e(s) = G_w(\omega, K)w(s) + G_v(\omega, K)v(s) \tag{28}$$

where the transfer matrices $G_w(\omega, K)$ and $G_v(\omega, K)$ are defined as: $G_w(\omega, K) = (sI - (A(\omega) - KC_i))^{-1}B_w$, $G_v(\omega, K) = -(sI - (A(\omega) - KC_i))^{-1}D_v$. Minimal sensitivity to parametric uncertainties at any speed ω can be achieved by minimizing the mean value of the $\mathcal{H}_\infty$ norm of the transfer matrix $G_w(\omega, K)$. However, this requirement can also lead to an increase of sensitivity to noise. It is then also necessary to limit the influence of noise. This can be done by imposing that the mean value of the $\mathcal{H}_2$ norm of the transfer matrix $G_v(\omega, K)$ is smaller or equal than a given value γ. Finally, a robustly stable flux observer, with, for all speeds ω, a small sensitivity to parametric uncertainties and noises measurement, can be obtained by solving the following mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization problem:

$$\begin{aligned} \text{minimize} \quad & \frac{1}{\bar{\omega} - \underline{\omega}} \int_{\underline{\omega}}^{\bar{\omega}} \|G_w(\omega, K)\|_\infty d\omega \\ \text{subject to:} \quad & \frac{1}{\bar{\omega} - \underline{\omega}} \int_{\underline{\omega}}^{\bar{\omega}} \|G_v(\omega, K)\|_2 d\omega \le \gamma \\ & \arg\max_i \mathrm{Re}\{\lambda_i(A(\underline{\omega}) - KC_i)\} \le \lambda_{min} \\ & K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ k_2 & k_1 & k_4 & k_3 \end{bmatrix}^T \end{aligned} \tag{29}$$

where K is the structured matrix of decision variables, $\lambda_i(.)$ denotes the i^{th} eigenvalue of the matrix passed in argument.

The parameter γ, is used to trade off between robustness to parametric uncertainties and noise sensitivity.

To simplify the resolution of (29), the integrals (i.e. the mean values) can be approximated by a finite sum by discretizing the rotor speed domain:

$$\begin{aligned} \text{minimize} \quad & \frac{1}{N_\omega}\sum_{i=1}^{N_\omega}\|G_w(\omega_i,K)\|_\infty \\ \text{subject to:} \quad & \frac{1}{N_\omega}\sum_{i=1}^{N_\omega}\|G_v(\omega_i,K)\|_2 \le \gamma \\ & \arg\max_i \mathrm{Re}\{\lambda_i(A(\underline{\omega})-KC_i)\} \le \lambda_{min} \\ & K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ k_2 & k_1 & k_4 & k_3 \end{bmatrix}^T \end{aligned} \tag{30}$$

with $\omega_1 = \underline{\omega}$ and $\omega_N = \bar{\omega}$.

D. Numerical experiment

In this numerical application, the following parameters of the induction machine have been used: $L_s = L_r = 0.5$H, $L_{sr} = 0.45$H, $Rs \in [0.75, 1.25]$, $Rr \in [0.5, 1.5]$ and $\omega \in [-100, 100]$. Rotor flux estimation can be done using (21) as a real time simulation model with nominal parameters. The sensitivity of this estimator to parametric uncertainties can then be evaluated by calculating $\max_\omega \|G_w(\omega,0)\|_\infty$, which gives: 1.31.

Now we can improve this result by solving problem (30). This problem has been solved using the Kalman optimization method with the following user defined parameters: $N = 50$, $N_\xi = 5$, $\alpha = 0.5$, $\gamma = 7$, $\lambda_{min} = -1.25$. The following observer gain has been found:

$$K^* = \begin{bmatrix} 62.060 & -7.357 & -2.261 & 0.291 \\ -7.357 & 62.060 & 0.291 & -2.261 \end{bmatrix}^T$$

The sensitivity of the resulting flux observer to parametric uncertainties is then evaluated by $\max_\omega \|G_w(\omega,K^*)\|_\infty$, which gives: 0.75. This result shows the superiority of the proposed flux observer over the real time simulation approach.

The performance of the proposed robust observer has been also compared with the classical Kalman estimator computed for $\omega = 50$rad/s, $Q = I_4$ and $R = I_2$, where Q and R are, respectively, the variance-covariance matrices of the process and measurement white noise. The following Kalman estimator has been found using the MatLab command `kalman`:

$$\begin{aligned} \dot{\hat{x}}(t) = & \begin{bmatrix} -26.1 & 0 & 18.95 & 947.4 \\ 0 & -26.1 & -947.4 & 18.95 \\ 1.266 & 0.1747 & -2 & -100 \\ -0.1747 & 1.266 & 100 & -2 \end{bmatrix}\hat{x}(t) + \\ & \begin{bmatrix} 10.53 & 0 & 7.05 & 0 \\ 0 & 10.53 & 0 & 6.95 \\ 0 & 0 & -0.37 & -0.17 \\ 0 & 0 & 0.16 & -0.37 \end{bmatrix}\begin{bmatrix} u_\alpha(t) \\ u_\beta(t) \\ i_\alpha(t) \\ i_\beta(t) \end{bmatrix} \\ \hat{y}_i(t) = & C_i\hat{x}(t) \end{aligned} \tag{31}$$

Figure 2 shows the flux modulus (*i.e.*, $\phi = \sqrt{\varphi_\alpha^2+\varphi_\beta^2}$) estimation obtained with the robust observer and the Kalman estimator. These results have been obtained with $u_\alpha = 100\sqrt{2}\sin(50t)$, $u_\beta = 100\sqrt{2}\sin(50t-\pi/2)$, $\hat{x}_K(0) = [1\ 1\ 1\ 1]^T$ and $\hat{x}_R(0) = [2\ 2\ 2\ 2]^T$, where $\hat{x}_K(0)$ and $\hat{x}_R(0)$ are, respectively the initial state vector of the Kalman estimator and the initial state vector of the robust observer. As shown figure 2, the the steady-state estimation error obtained with the robust observer is lower than that obtained with a standard Kalman estimator.

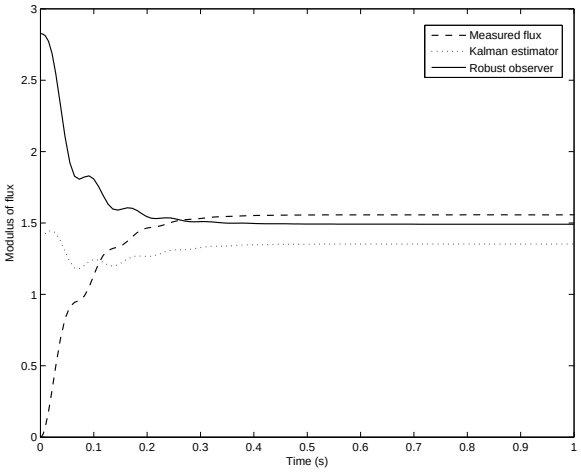

Fig. 2. Flux modulus estimation.

IV. Design of spiral inductors on silicon

In the sequel, we first introduce a well-accepted inductor model able to take into account the losses via parasitic resistances and capacitances. On the basis of this model, the optimal design of an on chip inductor is realized by using the Kalman optimization method.

A. Inductor model

Figure 3 shows the layout for square inductors, some other shapes can be used such as hexagonal, octagonal, or circular. For a given shape, an inductor is completely specified by the number of turns n, the turn width w, the turn spacing s, the inner diameter d_{in} and the outer diameter d_{out} (see figure 3). These parameters are typically the design variables of the inductor. Indeed, the inductance depends upon the geometry of the inductor, and so, for a desired inductance we have to determine the values of the layout parameters. But this is not sufficient, because at high frequencies (i.e. in the Ghz range), some complicated losses mechanisms must be taken into account to make a realistic design.

Figure 4(a) illustrates the basic structure of a planar spiral inductor on silicon. It consists of a metal trace manufactured by low-resistivity metals such as aluminium, copper, gold or silver. The metal spiral is mounted on silicon dioxide layer which acts as insulation between the metal trace and the silicon substrate. Figure 4(a) also highlights the parasitic resistances and capacitances which are introduced to model the losses. The corresponding electrical model of the spiral inductor on silicon is presented in figure 4(b), see [20] and [13] for a detailed derivation. This model takes into account the parasitic resistances and capacitances responsible of the

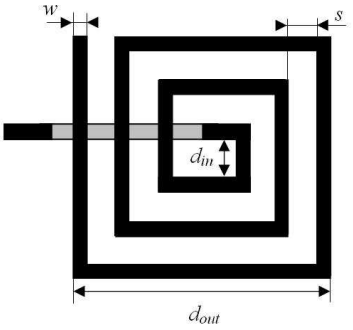

Fig. 3. Square inductor layout.

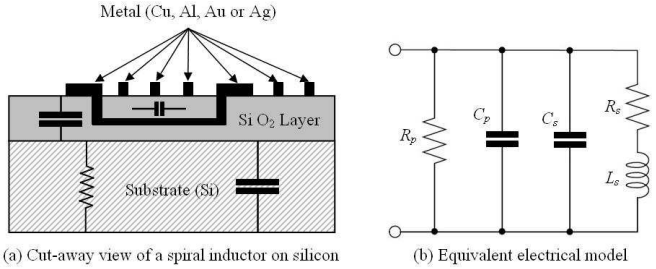

Fig. 4. Structure of an inductor on silicon and equivalent electrical model.

losses in the structure. The inductance L_s, and the resistances and capacitances R_s, C_s, R_p, C_p are defined as follows:

$$\begin{array}{l} L_s = k_1 n^2 z(d_{in}, d_{out}),\ \ R_s = k_2 n(d_{in} + d_{out})/w, \\ C_s = k_3 nw^2,\ R_p = 2k_7/(nw(d_{in} + d_{out})), \\ C_p = (k_8 + k_9)nw(d_{in} + d_{out})/2 \end{array} \tag{32}$$

The function $z(d_{in}, d_{out})$ and the constants k_1, k_2, k_3, k_7, k_8 and k_9 are given by:

$$\begin{array}{l} z(d_{in}, d_{out}) = c_1(\ln(c_2/r) + c_3 r + c_4 r^2), \\ r = (d_{out} - d_{in})/(d_{out} + d_{in}),\ k_1 = 2\pi 10^{-7}, \\ k_2 = \eta\rho/(d(1 - e^{-t/\delta})),\ \eta = c_5 \tan(\pi/c_5), \\ \delta = \sqrt{5 \times 10^6 \rho/(\pi\omega)},\ k_3 = \epsilon_{ox}/t_{ox,M_1-M_2}, \\ k_4 = \eta\epsilon_{ox}/(2t_{ox}),\ \ k_5 = \eta C_{sub}/2,\ \ k_6 = 2/(\eta G_{sub}) \\ k_7 = 1/(\omega^2 k_4^2 k_6) + k_6(k_4 + k_5)^2/k_4^2, \\ k_8 = k_4/(1 + \omega^2(k_4 + k_5)^2 k_6^2) \\ k_9 = k_4\omega^2(k_4 + k_5)k_5 k_6^2/(1 + \omega^2(k_4 + k_5)^2 k_6^2) \end{array} \tag{33}$$

where the parameters c_1, c_2, c_3, c_4, c_5 depend upon the shape of the inductor (square, hexagonal, octagonal or circular); the parameters ρ, t, ϵ_{ox}, t_{ox}, t_{ox,M_1-M_2}, C_{sub}, G_{sub} are technology dependent, and ω is the working frequency of the inductor.

The performance of an inductor is measured by its quality factor Q, which is limited by the parasitics. This quantity is defined as the ratio of peak magnetic energy minus peak electric energy to energy dissipated in the inductor see [20]:

$$Q = \frac{\omega L_s}{R_s} \frac{R_p \left[1 - \frac{R_s^2(C_s + C_p)}{L_s} - \omega^2 L_s(C_s + C_p)\right]}{R_p + \left[\left(\frac{\omega L_s}{R_s}\right)^2 + 1\right] R_s} \tag{34}$$

An inductor is at self-resonance when the peak magnetic and electric energies are equal. Therefore, Q vanishes to zero at the self-resonance frequency ω_{sr} i.e.:

$$\frac{R_s^2(C_s + C_p)}{L_s} + \omega_{sr}^2 L_s(C_s + C_p) = 1 \tag{35}$$

Above the self-resonance frequency, no net magnetic energy is available and thus it is generally required that $\omega_{sr} \geqslant \omega_{sr,min}$, where $\omega_{sr,min}$ is the desired minimal self-resonance frequency.

B. Formulation of the optimization problem

For a required value L_{req} of the inductance, the optimization consists in determining the values of the layout parameters (i.e n, w, s, d_{out} and d_{in}) which maximizes the quality factor while ensuring the desired minimal self-resonance frequency $\omega_{sr,min}$. In addition some geometry constraints must be added such as: a minimum turn width w_{min}, a minimum spacing s_{min}, a minimum inner diameter $d_{in,min}$ and a maximum outer diameter $d_{out,max}$ which limit the inductor area. The design variables d_{in} and d_{out} are not independent and are related to the other design variables by the expression $d_{in} + 2(n-1)s + 2nw = d_{out}$. Since s is typically small compared to d_{in}, d_{out} and w, we can recast this equality constraint as the inequality constraint: $d_{in} + 2n(w+s) \leqslant d_{out}$. The optimal design problem of the inductor can then be formulated as:

$$\begin{array}{ll} \text{maximize} & Q \\ \text{subject to} & L_s = L_{req} \\ & \omega_{sr} \geqslant \omega_{sr,min} \\ & d_{in} + 2n(w + s) \leqslant d_{out} \\ & s \geqslant s_{min},\ w \geqslant w_{min} \\ & d_{in} \geqslant d_{in,min},\ d_{out} \leqslant d_{out,max} \end{array} \tag{36}$$

C. Numerical experiments

Problem (36) has been solved using the Kalman optimization method, the results thus obtained were then validated using the field solver ASITIC. In our experiments, the following parameters have been used:

$$\begin{array}{l} c_1 = 1.27,\ c_2 = 2.07,\ c_3 = 0.18,\ c_4 = 0.13,\ c_5 = 4, \\ \rho = 2 \times 10^{-8}\Omega\text{m},\ t = 10^{-6}\text{m},\ \omega = 3\pi \times 10^9 \text{rad/s}, \\ \epsilon_{ox} = 3.45 \times 10^{-11}\text{F/m},\ t_{ox} = 4.5 \times 10^{-6}\text{m} \\ t_{ox,M_1-M_2} = 1.3 \times 10^{-6}\text{m},\ C_{sub} = 1.6 \times 10^{-6}\text{F/m}^2, \\ G_{sub} = 4 \times 10^4 \text{S/m}^2,\ s_{min} = w_{min} = 1.9 \times 10^{-6}\text{m}, \\ d_{in,min} = 10^{-4}\text{m},\ d_{out,max} = 4 \times 10^{-4}\text{m} \\ \omega_{sr,min} = 5\pi \times 10^9 \text{rad/s},\ L_{req} = 26 \times 10^{-9}\text{H}, \\ N = 50,\ N_\xi = 5,\ \alpha = 0.5. \end{array}$$

The solutions found via HKA is given by: $d_{out} = 236 \times 10^{-6}$m, $d_{in} = 113.8 \times 10^{-6}$m, $w = 4.4 \times 10^{-6}$m, $s = 1.9 \times 10^{-6}$m, $n = 10$. Using ASITIC as a verification tool (i.e. with the layout parameters found via HKA, the field solver ASITIC is used to determine the corresponding L, Q and ω_{sr}), we get the results shown in Table II. As we can see, the results obtained using HKA are very close to those predicted using ASITIC.

TABLE II
VERIFICATION OF THE SOLUTION FOUND WITH ASITIC.

	L (nH)	Q	ω_{sr} (GHz)
HKA	26	3.53	≥ 2.5
ASITIC	25.82	3.59	2.55

V. CONCLUSION

In this paper, a new optimization algorithm, called heuristic Kalman Algorithm (HKA), was presented. The main characteristic of the HKA is to explore, via a Gaussian pdf, the search space. This exploration is directed by an appropriate adjustment of the pdf parameters in order to converge to a near-optimal solution with a small variance. To this end, a measurement process followed by a Kalman estimator was introduced. The role of the Kalman estimator is to combine the prior pdf function with the measure to give a new pdf function for the exploration of the search space.

HKA has been applied in two domains of industrial electronics, namely the design of a robust flux estimator of an induction machine and the optimal design of on chip-spiral inductors. These design problems have led to the formulation of non-convex constrained optimization problems which are known to be difficult to deal with. It has been shown that HKA can be used to solve this kind of problem in a direct way without requiring too many user defined parameters unlike other stochastic methods.

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