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## **RESEARCH ARTICLE**

#### A new stochastic inverse identification of the mechanical properties of human skin

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# **RESEARCH ARTICLE**

## A new stochastic inverse identification of the mechanical properties of human skin

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Abstract

The study of the human skin mechanical properties is a key point to better understand surgery, skin ageing and pathologies. As the skin is a living tissue, it must be studied *in vivo*, hence analytical solutions are really difficult to obtain. In this study, a new stochastic inverse method for the identification of its mechanical properties is proposed. The developed optimization method is first presented. It is based on an iterative stochastic approach which ensures the identification of a global extremum. The suction actual case study is then analyzed through comparisons between experimental data and finite element models of this test. Only the elastic components of the skin are considered here. The solutions for the Recursive Least Squares and Gauss-Newton's problems are finally compared with the proposed approach to conclude on this study and to briefly present our future works.

Keywords: human skin, in vivo, suction test, finite element, inverse method, stochastic approach

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# 1. Introduction

The mechanical properties of living tissues are of potential interest in the identification of certain diseases, for assessing therapeutic intervention, or for predicting the effect of trauma. Skin is a complex medium which is made up of three layers: the epidermis, the dermis and the hypodermis (see Figure 1). Due to the specific structure of the dermis, the skin presents a nonlinear viscoelastic anisotropic quasi incompressible mechanical behaviour which has to be studied in vivo [1]. To identify the skin mechanical properties, complex numerical models which lead to multi-parameter optimization problems are usually studied [2]. When considering a large number of parameters to be identified, the convergence of standard optimization methods [3] is difficult to obtain [4, 5]. The optimization algorithms thus need to reach the solution despite the local minima and stabilization problems. Moreover they are known to be very sensitive to the initial set of values and require specific stabilization procedures (e.g. linesearch methods [6]). Finally, as clinical studies are performed with a large number of measurements, the mechanical parameters need to be identified rapidly.

Hence the developed stochastic method aims at identifying the mechanical properties of the skin. The suction deformation test which is performed on the volar aspect of the forearm of a subject is first described. The obtained experimental results are then compared to those related to finite element models of this test through an optimization computation. This process is used so as to minimize the difference between the experimental and the numerical data. To underline the usefulness of the proposed method, comparisons with standard Gauss-Newton's and Recursive Least Squares methods are drawn for a blind test. It consists in using simulated curves as if they were experimental ones. These methods are then compared for the identification of an actual suction experiment. The reliability of the proposed approach and its sensitivity to the random samples are finally discussed to draw some conclusions and to briefly present our future works.



FIGURE 1: The multi-layered structure of the skin

# 2. Method

The proposed method is divided into three stages. First the suction experiment is performed and modeled through the finite element method. A numerical data base which is made up of simulated curves that are obtained for different values of the mechanical parameters is then created.

# 2.1. The suction experiment

The in vivo suction test [7, 8] consists in applying a negative pressure at the skin surface. In our case study, this test is performed on the volar aspect of the forearm skin, using a Cutometer CM570 (Courage & Khazaka, Cologne, Germany). The skin is sucked into a cylindrical aperture of 6mm diameter forming a dome whose deflection M is recorded for each step of pressure  $p_t$  (see Figure 2). Figure 3 presents an actual curve which was obtained for a pressure up to 100mbar, applied at a rate of 20mbar.s<sup>-1</sup>. The experimental curves generally contain 50 to 100 points which is assumed to be sufficient to perform the inverse computation. As an analytical solution for the suction problem related to complex behaviour laws is difficult to obtain, finite element numerical models are created.



FIGURE 2: The suction experiment principle



FIGURE 3: Experimental suction curve obtained on the volar aspect of the forearm (thickness  $\approx 1$  mm)

#### 2.2. The finite element model

The numerical simulations are performed with the SYSTUS<sup>™</sup> finite element software. To model a complex structure such as skin, several assumptions need to be made. The skin is considered to be a homogeneous medium, its structural response is only considered here. Due to the low stress rate, the mass inertial effects are neglected and computations are thus performed using a quasi-static calculation. The geometry is considered to be axisymmetrical (see Figure 2) and the skin was modeled as a single layer whose thickness is the one corresponding to the epidermis and the dermis. It is measured using a high frequency (20MHz) ultrasound device (Dermcup, ATYS Medical) [9]. Indeed, results from literature clearly state that, due to the low elastic properties of the hypodermis [10, 11], the effects of sub-dermal structures can be neglected [8, 12, 13]. The resolution of the ultrasound device [9] and the papillary structure of the epidermis and the dermis [1], prevent from distinguishing these layers. Hence, they are considered as melted. Moreover, this leads to simplify the calculations and to avoid finite element numerical instabilities [14]. In contrast to studies that account for the viscoelastic and the anisotropic behaviour of the skin [15], our approach consists in identifying its isotropic elastic mechanical properties. Second order elements with a reduced integration scheme so as to avoid volumetric locking are used for the mesh. As adhesive tapes are used to paste the suction device to the skin, the nodes corresponding to this interface are assumed to be restrained through axial and lateral directions (see Figure 2). An identical restriction was applied to the lateral displacements of the symmetrical axis nodes to properly computer the 2D calculations.

The suction test usually involves large deformations and large displacements in the skin. Hence, the behavior law and the formulation that are used should account for the geometrical nonlinearity of the material [16]. As standard hyperelastic behaviors (e.g. neo-Hookean or Mooney-Rivlin potentials) cannot be used to model the skin [16, 17], a specific compressible law, based on an extended neo-Hookean potential [4, 13] was developed (see Equation (1)).  $X_1$  and  $X_2$  are the elastic parameters,  $J_1$  and  $J_2$  denote the first and second reduced invariants of the right Cauchy-Green deformation tensor C (they can be expressed as a function of the standard invariants  $I_1$  and  $I_2$  of C see Equations (2.a) and (2.b)), J = det(F), where F is the deformation gradient, J is the volume ratio and  $\kappa$  the compressibility constant of the media. For small strains, a relationship can be drawn between the compressibility constant and Poisson's ratio v (see Equation (3)). Three mechanical parameters thus need to be identified:  $X_1$ ,  $X_2$  and v.

$$W = X_1 (J_1 - 3) + X_2 (J_1 - 3) (J_2 - 3) + \frac{\kappa}{2} (J - 1)^2$$
(1)

$$J_1 = I_3^{-1/3} I_1$$
 (2.a)

$$I_2 = I_3^{-2/3} I_2 \tag{2.b}$$

$$\kappa = \frac{6(1-\upsilon)X_1}{(1+\upsilon)(1-2\upsilon)} - 8/3 X_1 \tag{3}$$

#### 2.3. The principle of the inverse method

The main originality of the proposed method lies in the construction of a pre-calculation data base: the simulated space (see Figure 4.a) [5]. The results of several numerical calculations (deflection as a function of the applied pressure), relating to the different combinations of the mechanical parameters to be determined, are collected in this data base  $\Omega$ . Indeed, clinical studies are performed with a large number of measurements. With the use of standard inverse methods, a finite element computation is required for each identification step. Redundant solutions to the numerical models may thus be calculated. In the present procedure, all the finite element models are performed only once. Furthermore, such experimentation is usually performed by medical practitioners, hence our method needs to be a FEM-free process (see Figure 4.b).



FIGURE 4: (a) Simulated space defined for a compressible neo-Hookean law (2 mechanical parameters:  $X_1$  and v). (b) The proposed approach.

The simulated space  $\Omega$  which proposes the variation of the deflection as a function of the mechanical parameters for each step of pressure is defined by Equation (4), where  $\mathbf{x} = \begin{bmatrix} X_1 & X_2 & v \end{bmatrix}^T$ , and, for *u* points of measurement,  $\mathbf{p} = \begin{pmatrix} p_0 & \cdots & p_u \end{pmatrix}$ . Its variation field is:  $X_1(\text{Mpa}) \in [0.01; 0.1]$  per step of 0.01MPa,  $X_2(\text{Mpa}) \in [0.1; 4.9]$  per step of 0.3MPa,  $v \in [0.2; 0.48]$  per step of 0.04 and  $p(\text{mbar}) \in [0; 100]$  per step of 2mbar.

$$\Omega_{s}: (\mathbf{x}, \mathbf{p}) \mapsto \Omega_{s}(\mathbf{x}, \mathbf{p}) \tag{4}$$

The forward problem thus consists in identifying the parameters  $X_1^*$ ,  $X_2^*$  and  $v^*$  that minimize:

$$\mathbb{S}^{g}: (\mathbf{x}) \mapsto \frac{1}{2} \mathbf{D}(\mathbf{x})^{r} \mathbf{D}(\mathbf{x})$$
(5)

where, for iteration j and u points of measurement M (see Figure 3),  $\mathbf{D}(\mathbf{x}^{j})^{T} = [d_{1}(\mathbf{x}^{j}) \dots d_{u}(\mathbf{x}^{j})]$ , with  $d_{i}(\mathbf{x}^{j}) = M(\mathbf{x}^{*}, p_{i}) - \mathbf{\Omega}_{s}(\mathbf{x}^{j}, p_{i}), \mathbf{x}^{*} = [X_{1}^{*} \quad X_{2}^{*} \quad \upsilon^{*}]^{T}$  and  $t = 1, \dots, u$ .

During the calculation, non-simulated values are identified. These are simply calculated with cubic Lagrange's interpolations of the existing data.

### 2.4. The fundamentals of the stochastic algorithm

In this paper a heuristic method to solve the following optimization problem is proposed [5]:

$$\mathbf{x}^* = \arg\min_{\mathcal{I}} g(\mathbf{x}) \tag{6}$$

It does not require the derivatives of the cost function  $\mathcal{I}$  nor its mathematical expression (i.e. the values of  $\mathcal{I}$  are here obtained through numerical simulations of the suction experiment). The set  $\mathbf{x} \subset \mathbf{R}^n$  represents the admissible solutions. To solve this problem, the principle depicted in Figure 5 is considered. *N* vectors that are uniformly distributed between the bounds  $\underline{\mathbf{x}} = \begin{bmatrix} \underline{x}_1 & \cdots & \underline{x}_n \end{bmatrix}^T$  and  $\overline{\mathbf{x}} = \begin{bmatrix} \overline{x}_1 & \cdots & \overline{x}_n \end{bmatrix}^T$  are first determined, at each iteration, through a uniform random generator:

$$\mathbf{X}^{j} = \left\{ {}_{1}\mathbf{x}^{j}, {}_{2}\mathbf{x}^{j}, \dots, {}_{N}\mathbf{x}^{j} \right\}$$
(7)

where,  $_{k}\mathbf{x}^{j}$  is the k<sup>th</sup> vector generated at the iteration  $j: _{k}\mathbf{x}^{j} = \begin{bmatrix} _{k}x_{1}^{j} & \cdots & _{k}x_{n}^{j} \end{bmatrix}^{r}$ , and  $_{k}x_{l}^{j}$  is the  $l^{\text{th}}$  component of  $_{k}\mathbf{x}^{j}$  with  $l = 1, \dots, n$ . These random vectors are then used to compute their related cost functions  $\mathcal{I}$  (see Equation (5)) and they are finally sorted according to their increasing values i.e.:

$$\mathcal{J}(\mathbf{x}^{j}) < \mathcal{J}(\mathbf{x}^{j}) < \cdots < \mathcal{J}(\mathbf{x}^{j})$$
(8)

The principle of the algorithm is to modify the bounds  $\underline{x}$  and  $\overline{x}$  of the random generator until the minimum of the cost function is reached. Obviously, this minimum is characterized by a difference between the bounds close to zero (i.e. the euclidian norm of the difference  $\overline{x} - \underline{x}$  is small). Hence, the given problem is to find how to modify the bounds of the random generator to converge to the optimum.

To solve this problem the  $N_b$  best candidates are considered (i.e.  $N_b$  candidates that present the lowest cost function). The solution is assumed to be dissimilar from the bounds of the calculation space. For the iteration *j*, the bounds  $\bar{x}$  and  $\bar{x}$ , of the random generator are then evaluated as follows:

$$\begin{cases} \underline{x}_{l} = \min_{1 \le k \le Nb} k x_{l}^{j} \\ \overline{x}_{l} = \max_{1 \le k \le Nb} k x_{l}^{j}, \quad l = 1, \dots, n \end{cases}$$
(9)

Nevertheless, the final result which is obtained is not an exact but a near optimum that is closely related to number of samples N. The minimum number of samples required to obtain a reliable optimum, must be chosen to as [16, 17]:

$$N \ge \ln(1-\rho) / \ln(1-e) \tag{10}$$

where  $\rho$  is the confidence ( $\rho$  must be close to 1 e.g.  $\rho = 0.99$ ) and *e* is an accuracy (*e* must be close to 0 e.g. e = 0.01).



FIGURE 5: Principle of the developed algorithm.

One can note that this method can be modified through a Kalman procedure for an optimal updating of the statistical characteristics of the random generator [19]. Nevertheless, only the standard approach (see Figure 5) is considered in this paper.

# 2.5. Implementation of the stochastic approach

The fundamentals of the stochastic algorithm were implemented in three stages [5]. First, the maximal and minimal boundary parameters  $\underline{\mathbf{x}}^{0} = \begin{bmatrix} \underline{X}_{1}^{0} & \underline{X}_{2}^{0} & \underline{v}^{0} \end{bmatrix}^{T}$  and  $\overline{\mathbf{x}}^{0} = \begin{bmatrix} \overline{X}_{1}^{0} & \overline{X}_{2}^{0} & \overline{v}^{0} \end{bmatrix}^{T}$  are defined. They usually correspond to the boundary values of the simulated space (Figure 6 step 0). *N* values of this domain are then randomly chosen and, for each iteration *j*, their related solutions  $\Omega_{s}(_{k}\mathbf{x}^{j},\mathbf{p})$  with  $k = 1, \dots, N$  are calculated thanks to the simulated space (Figure 6 step 1). They are finally sorted according to the quadratic error  $E^{j}$  which is defined in Equation (11). The minimal and maximal values of the  $N_{b}$  firsts sorted results are finally selected for each mechanical parameter, to be the starting point of an other iteration (Figure 6 step 2). This process iterates till it satisfies a stop criterion  $C^{j} = |E^{j-1} - E^{j}|$ . The identified parameters are the ones corresponding to the final iteration best cost function.





FIGURE 6: Implementation of the stochastic approach for N = 67,  $N_b = 8$  and two parameters to be identified:  $Y_1$  and  $Y_2$ .

### 3. Results

### 3.1. Theoretical results

To check the relevance of the stochastic algorithm (*STO*) and to assess the influence of the assumptions that are made, a blind test was performed. This test consists in using curves extracted from the simulated space as if they were experimental ones. The three known parameters,  ${}^{bi}X_1 = 0.03$ MPa,  ${}^{bi}X_2 = 1$ MPa and  ${}^{bi}v = 0.32$ , are considered.

To underline the usefulness of the proposed method, comparisons are drawn with the results of the standard Gauss-Newton (GN) [3] and Recursive Least Squares (RLS) methods [20]. For the iteration j, the GN scheme is described by:

$$\mathbf{x}^{j+1} = \mathbf{x}^j + \mathbf{W}^j \tag{12.a}$$

For linesearch techniques it can be re-written to as:

$$\mathbf{x}^{j+1} = \mathbf{x}^j + \alpha^j \mathbf{W}^j \tag{12.b}$$

where  $\alpha^{j}$  is a scalar which varies according to a 1D minimization of the problem at iteration *j*. Nonetheless the standard approach is only considered in this paper. The use of the  $\alpha^{j}$  parameter is mentioned in the discussion.  $\mathbf{W}^{j}$  is the descent direction which is defined through the jacobian matrix  $\mathbf{F} = \partial \mathbf{D}(\mathbf{x})/\partial \mathbf{x}$ :

$$\mathbf{W}^{j} = \left(\mathbf{F}^{j^{T}}\mathbf{F}^{j}\right)^{-1}\mathbf{F}^{j^{T}}\left(M\left(\mathbf{x}^{*},\mathbf{p}\right) - \mathbf{\Omega}_{s}\left(\mathbf{x}^{j},\mathbf{p}\right)\right)$$
(13)

The *RLS* algorithm is based on the computation of D(x) for each step of measurement  $p_t$ . To improve its reliability, the *RLS* technique was developed thanks to a forgetting parameter approach ( $\lambda_0 = 0.9$ ) [4, 20]. The time update equations are defined by:

$$\mathbf{x}^{t+1} = \mathbf{x}^{t} + \mathbf{A}^{t} \left( M \left( \mathbf{x}^{*}, p_{t} \right) - \boldsymbol{\Omega}_{s} \left( \mathbf{x}^{t}, p_{t} \right) \right)$$
(14)

where  $\mathbf{A}^{t} = \mathbf{P}^{t} \mathbf{H}^{t} (\mathbf{H}^{t} \mathbf{P}^{t} \mathbf{H}^{t} + \lambda_{t} \mathbf{I}^{(1)})^{-1}$ , with  $\mathbf{H}^{t} = \partial \Omega_{s}(\mathbf{x}, \mathbf{p}) / \partial \mathbf{x} \Big|_{p_{t}, \mathbf{x}^{t}}$ ,  $\mathbf{I}^{(y)}$  is the yth order identity matrix, and  $\mathbf{P}^{t}$  is the covariance matrix which is here initialized to  $\mathbf{P}^{t} = \mathbf{I}^{(3)}$ . From step t to step t+1, this matrix is updated through:

$$\mathbf{P}^{t+1} = \left(\mathbf{P}^{t} - \mathbf{A}^{t} \mathbf{H}^{t} \mathbf{P}^{t}\right) \lambda_{t}^{-1}$$
(15)

and  $\lambda_r$  using the following equation [20]:

$$\lambda_{r+1} = \lambda_0 \lambda_r + (1 - \lambda_0) \tag{16}$$

This method was modified through a stabilization process which consists in running the calculations several folds on the same curve [4]. Hence, one iteration of the algorithm is then related to one run of the experimental curve and thus u step of calculation are made.

For both the *GN* and *RLS* methods, the initial parameters are equal to the minimal boundary values of the simulated space. The calculations were performed on a Pentium core-duo 1.6GHz, RAM 1Go.

Method	$^{id}X_{1}\left( \mathrm{MPa} ight)$	$^{id}X_2(\mathrm{MPa})$	$^{id} v$	CPUT(s)	<i>E</i> (µm)
GN	0.03	1	0.32	3.1	0
RLS	0.03	1	0.32	10.5	0
STO $N=10^2$	0.029	0.93	0.34	2.5	0.09
STO $N=10^3$	0.03	1.05	0.30	11.1	0.011
STO $N=10^4$	0.03	1.01	0.32	58	0.002

TABLE 1: Identified values for a blind test.  ${}^{bt}X_1 = 0.03$  MPa ,  ${}^{bt}X_2 = 1$  MPa ,  ${}^{bt}v = 0.32$  ,  $C = 10^{-5}$  µm , and  $N_b = 20$ . *CPUT* is related to the computation time, *E* is the final quadratic error which is defined in Equation (11).

Table 1 presents the obtained results ( ${}^{id}X_1$ ,  ${}^{id}X_2$  and  ${}^{id}v$ ) for the three different approaches. Random samples of  $N = 10^2$ ,  $N = 10^3$  and  $N = 10^4$  are considered for the stochastic method. The convergence criterion  $C^{j} = |E^{j-1} - E^{j}|$  is equal to  $10^{-5}\mu$ m. One can remark that the *GN* and *RLS* algorithms reach the required mechanical parameters whereas the stochastic technique is unable. For each case study estimates are obtained. Nevertheless the error presents low values and the identified parameters are in good agreement with the required ones. The number *N* of the random samples hardly influences the results whereas it strongly affects the computation time (*CPUT*). Hence, a compromise between *CPUT* and the calculation accuracy could be easily found. One can note that, according to Section 2.4, for an infinite number of samples, the required solution will be identified.



FIGURE 7: Influence of the  $N_b$  sorted values on the obtained results. The results are obtained while considering  ${}^{bt}X_1 = 0.03$  MPa ,  ${}^{bt}X_2 = 1$  MPa ,  ${}^{bt}v = 0.32$  ,  $C = 10^{-5}$  µm and  $N = 10^3$ .

One can also study the influence of the  $N_b$  sorted values on the obtained results. Figure 7 plots the variations of the ratio  ${}^{id} \mathbf{x}/{}^{bi}\mathbf{x}$  for each considered parameter. For  $N_b < 10$ , the results show a significant identification error whereas for higher values of this criterion, the identified parameters remain stable. One can note that the accuracy of the problem seems to decrease for  $N_b > 50$ . Moreover, the number of iterations necessary to reach the convergence ( $C = 10^{-5} \mu m$ ) linearly increases with  $N_b$  (see Figure 8.a). For actual cases study,  $N_b$  was thus chosen equal to 20. Nevertheless this point has to be discussed according to the number of random samples. Figure 8.b plots the influence of the ratio  $N/N_b$  on the number of iteration. For every study cases and for a similar identification error, it decreases as  $N/N_b$  increases. Nevertheless, this ratio should be chosen with care as the number or random samples strongly affects the computation time (e.g. *CPUT* = 3.1s with N = 500 and  $N_b = 5$ , and *CPUT* = 40.4s with N = 5000 and  $N_b = 50$ ).



FIGURE 8: Influence of the  $N_b$  sorted values on the number of iterations.  ${}^{bi}X_1 = 0.03$ MPa,  ${}^{bi}X_2 = 1$ MPa and  ${}^{bi}v = 0.32$ ,  $C = 10^{-5}\mu$ m. (a) Obtained results for  $N = 10^3$ . (b) Obtained results according to the ratio  $N/N_b$  for  $N = \begin{bmatrix} 10^2 & 2.5 \times 10^2 & 5 \times 10^2 & 7.5 \times 10^2 & 10^3 & 2.5 \times 10^3 & 5 \times 10^3 & 7.5 \times 10^3 & 10^4 \end{bmatrix}$  and  $N_b = \begin{bmatrix} 2 & 3 & 4 & 5 & 10 & 20 & 30 & 40 & 50 & 75 & 100 & 150 & 200 \end{bmatrix}$ .

## 3.2. Experimental study case

Table 2 presents the results corresponding to the experimental curve (Figure 3). For that study case, the stochastic approach shows the best results whatever the value of N is. One can note that even if N presents high values, the computation time can decrease if less iterations are needed to reach the convergence. Figure 9 plots a comparison between the experimental and the simulated deflections obtained for the identified parameters. It shows that the results are in agreement.

Method	$^{id}X_1$ (MPa)	$^{id}X_2(\mathrm{MPa})$	$^{^{id}}\nu$	CPUT(s)	E(µm)
GN	0.016	0.1	0.48	3.2	43
RLS	/	/	/	/	/
STO $N=10^2$	0.025	2.95	0.28	2.3	2
STO $N=10^3$	0.026	3.1	0.26	1.4	2
STO N=10 <sup>4</sup>	0.025	3.06	0.26	15.2	2

TABLE 2: Identified values for an actual case study,  $C = 10^{-5} \mu m$ , and  $N_b = 20$ . *CPUT* is related to the computation time, *E* is the final quadratic error which is defined in Equation (11).



## ♦ Experimental curve —— Identified curve

FIGURE 9: Comparison between the experimental and the identified curves obtained for  $N = 10^3$ ,  $C = 10^{-5} \mu m$ ,  $N_b = 20$ .

The *GN* algorithm stabilizes to the bounds of the simulated space and thus presents a high computation error. Indeed, such algorithms are known to be very sensitive to the initial set of values and to the problem nonlinearity. For initial values that are close to the stochastic identified ones, and for specific constant values of  $\alpha$  (e.g. 0.1, 0.01, see Equation (12.b)), the *GN* algorithm still diverges. Specific methods (e.g. Levenberg-Marquardt [21, 22]) should thus be used to successfully perform the identification.

The *RLS* results are more difficult to analyse. For  $\lambda_0 = 0.9$ , the calculation cannot stabilize for the specified convergence criterion, however, for higher values (i.e.  $C = 10^{-2} \mu m$  with  $C_j = |E_{j-1} - E_j|$ ) the convergence can be reached with a low computational error ( $E = 2.1 \mu m$ ). Indeed the RLS convergence shows oscillating values around the solution. These oscillations usually make the computation diverge (see Figure 10). Identical conclusions have been drawn for different values of  $\lambda_0$ . Figure 10 clearly states that the error is minimized around the 10<sup>th</sup> iteration. Due to the considered convergence criterion, the calculation still iterates and finally destabilizes until it reaches the bounds of the simulated space.

Several sets of initial values were tested to study the sensitivity of both the *GN* and the *RLS* algorithms to the computation initialization. In every case these algorithms do not reach satisfactory solutions. According to these conclusions linesearch techniques should therefore be used for both these computations whereas the stochastic approach does not need any.



FIGURE 10: Oscillating values of the *RLS* for a convergence criterion  $C = 10^{-5} \mu m$  and a forgetting parameter  $\lambda_0 = 0.99$ .



FIGURE 11: Mean values and standard deviations according to 50 cases study and different number of random values for the identified parameters  ${}^{id}X_1$ ,  ${}^{id}X_2$ ,  ${}^{id}v$ . The results are obtained while considering  $C = 10^{-5} \mu m$ ,  $N_b = 20$ ,  $N = 10^3$ .

The relevance of the stochastic method was tested for 50 computations which correspond to different random samples. Figure 11 shows the influence of *N* on the mean error and on the standard deviation of the final solutions for each calculation. Regarding to the proposed mean parameters, the standard deviation presents low values. As an example Figure 12 plots the identified parameters for  $N = 10^3$  according to each case study. As one would expect (see Section 2.4) an increase of *N* reduces the standard deviation. It would finally tend to 0 if *N* tends towards the infinity, as the global minimum is reached. One could also remark that the mean values obtained for  $N = 10^2$  are different than the ones related to  $N = 10^3$  and  $N = 10^4$ . This clearly states that the number of random samples should be chosen with care and according to the problem.



FIGURE 12: Identified mechanical parameters for the stochastic technique according to 50 random cases.  $C = 10^{-5} \mu m$ ,  $N_b = 20$ ,  $N = 10^3$ . Experimental study case.

This calculation was also successfully performed on 300 experimental curves (a thickness of the dermis equals to 1mm was considered for every individual [16]). Figure 13 presents the distribution of the identification errors according to theses study cases. The error mean value is about 1.6µm and the standard deviation equals 1.1µm which proves the accuracy of the proposed method. The most important error values (see Figure 13) are related to specific experimental curves that are probably related to measurement problems.



FIGURE 13: Distribution of the identification errors according to 300 experimental curves.  $C = 10^{-5} \mu m$ ,  $N_b = 20$ ,  $N = 10^{3}$ .

## 4. Discussion

In this article, a new stochastic approach which ensures the identification of a global minimum was developed. To set the theoretical background of this method, its fundamentals were first presented. A specific implementation of this algorithm was then proposed for the identification of the mechanical properties of the human skin. It consists in drawing comparisons through a data base between in vivo experimental results and the ones related to finite element models of the suction test. The proposed approach was then compared to the well-known Gauss-Newton and Recursive Least Squares optimization algorithms for a given blind test. This case study shows that good estimates can easily be identified whereas the exact solution requires an infinite number N of samples. Hence, for theoretical tests deterministic algorithms presents better results. Finally an actual case study was considered. The stochastic solution was successfully compared with the GN and RLS ones. In our case study, the GN identification usually reaches the boundary values of the simulated space and the RLS approach oscillates around the solution. This was also tested for several other calculations. In most of the cases, identical conclusions can be drawn. Both processes are known to be sensitive to the initial values and, without a specific linesearch technique, they may oscillate around local extrema or diverge. For each iteration, a specific optimization procedure is thus required. Moreover, these standard methods necessitate evaluate the derivatives of the cost function of the problem. Time and memory consuming calculations which are not required by the stochastic approach are thus performed.

The main advantages of the developed method lie in its relevance. Indeed, many parameters can be identified easily without neither computational divergence nor oscillating values. Furthermore, no initial set of values, which constitutes a critical stage for most of the optimization processes, is required. For the first step of the calculation, the bounds of the simulated space are simply considered. The stochastic method is theoretically able to reach the global minimum of the problem. However, this requires an infinite number N of random

samples, hence, the obtained results should be interpreted in terms of probability  $\rho$  and accuracy *e* (see Equation (10)). To improve the presented results, various approaches to choose the *N* random parameters were also analyzed. The Latin Hypercube Sampling (*LHS*) [23] was for example considered. However, each of these studied methods affects neither the convergence nor the identified parameters.

This method will be from now on extended to the identification of the mechanical parameters of the human skin through more complex behaviour laws and multi-layered models. The viscosity of the skin tissue should be discussed for more reliable results. Rheological models [15, 17, 24] or mixture approaches [25] generally present a large number of parameters to identify, moreover, one can note that identical observations are proposed by multi-layered analyses [13, 14]. Hence, the proposed approach allows discussing easily the uniqueness of the solution which is a key-point to draw reliable conclusions on the results.

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