On-line reliability prediction via dynamic failure rate model

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Abstract. This paper presents a new dynamic reliability approach which is able to take into account the various operating conditions under which the considered system has evolved (past and present). To this end, a differential model of failure rate combined with the state space representation of the process is proposed, as well as a parametrization procedure. A numerical example shows the practical applicability of the proposed approach.

Keywords: Reliability prediction, dynamic reliability models, state-space representation, operating functions.

1 Introduction

Traditional reliability methods are usually based on analysis of life-time data. These data can then be used for the parametrization of models reflecting population characteristics, under the same conditions, as those constituted in the data [3, 8, 2, 9, 7]. These models have proved their usefulness during design phases, but are of limited interest in operational phases, because, in this case, operational conditions are constantly changing. Indeed, a real system evolves under various operating conditions, and this has a direct impact on its reliability. Therefore, traditional reliability methods are not well adapted in estimating system reliability under dynamic operating conditions.

However, as we will see, the traditional static reliability methods can be extended to the dynamic case by taking into account the past degradations, due to the past operating conditions under which the process has evolved. This aspect is crucial in the operational reliability context.

From this point of view, the main objective of this paper is to give a model able to predict the reliability in real time, that is, able to take into account the history of process running. To this end, a differential model of failure rate combined with the state space representation of the process is proposed, as well as a parametrization procedure.

This approach is in contrast with the work presented in [4, 5] in which the reliability prediction is based on the assumption that there are some measurable variables reflecting directly the degradation of the system. This assumption is removed in this work. In the proposed approach all measurable variables are used in order to predict reliability resulting from past and present time-varying operating conditions.

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2 Process and dynamic failure rate model

We will deal with dynamical systems that are modeled by a finite number of coupled first-order ordinary differential equation

$$\begin{cases} \dot{x}_1(t) = f_1(x_1(t),..,x_{n_x}(t),u_1(t),...,u_{n_u}(t)) \\ \vdots \\ \dot{x}_{n_x}(t) = f_{n_x}(x_1(t),..,x_{n_x}(t),u_1(t),...,u_{n_u}(t)) \end{cases} \tag{1}$$

where $\dot{x}_i$, $i = 1,\cdots,n_x$, denotes the derivative of x_i with respect to the time variable t and u_1, u_2, $\cdots$, u_{n_u} are specified input variables. We call the variables x_1, x_2, $\cdots$, x_{n_x} the states variables. They represent the memory that the dynamical system has of its past. The f_i, $i = 1,\cdots,n_x$, are assumed to be known continuous functions. We usually use vector notation to write these equations in a compact form

$$\begin{array}{c} x(t) = [x_1(t) \ \cdots \ x_{n_x}(t)]^T,\, u(t) = [u_1(t) \ \cdots \ u_{n_u}(t)]^T \\ f(x(t),u(t)) = [f_1(x(t),u(t)) \ \cdots \ f_{n_x}(x(t),u(t))]^T \end{array} \tag{2}$$

and rewrite the n first-order differential equations as one n-dimensional first-order vector differential equation

$$\dot{x}(t) = f(x(t),u(t)) \tag{3}$$

we call (3) the state equation and refer to x as the state and u as the input. It can be noted that the input vector u may include both the control inputs and the exogenous or perturbations inputs. The global system consists of a given number of components and we are interested of estimating on-line the reliability of one or a group of particular components assumed to be crucial for the safety of the system. To this end, it is necessary to take into account the operating conditions because they have a direct influence on reliability. Of course, the operating conditions of a particular component, depend upon the input u and the state x around which the system evolves. In these conditions, a realistic reliability model must include the physical process variables i.e. the input u and the state x. However, this is not sufficient because operational conditions are constantly changing in time, and this has a direct impact on reliability. It is then also necessary to take into account the history of operating conditions under which the system has evolved. For this purpose, one can use a dynamical failure rate model of following general form :

$$\dot{\lambda}(t) = \phi(t,x(t),u(t)) \tag{4}$$

where λ represents the failure rate, ϕ a function of the time and of the operating conditions i.e. $(x(t),u(t))$. This model must be completed with the dynamical model describing the evolution in time of the operating conditions i.e. equation (3). The global dynamical failure-rate model is then given by :

$$\begin{cases} \dot{\lambda}(t) = \phi(t,x(t),u(t)) \\ \dot{x}(t) = f(x(t),u(t)) \end{cases} \tag{5}$$

A particular model of great importance for practical application is as follows :

$$\begin{cases} \dot{\lambda}(t) = t^{n(x(t),u(t))}\psi(x(t),u(t)) \\ \dot{x}(t) = f(x(t),u(t)) \end{cases} \tag{6}$$

where ψ and n are functions of operating conditions. For a constant operating condition denoted by (x_0,u_0), the integration of $\dot{\lambda}$ with respect to the time leads to the Weibull law :

$$\begin{cases} \lambda(t) = \dfrac{\beta(x_0,u_0)}{\eta(x_0,u_0)} \left(\dfrac{t}{\eta(x_0,u_0)}\right)^{\beta(x_0,u_0)-1} \\ \\ f(x_0,u_0) = 0 \end{cases} \tag{7}$$

with

$$\begin{cases} n(x_0,u_0) = \beta(x_0,u_0) - 2 \\ \psi(x_0,u_0) = \dfrac{\beta(x_0,u_0)(\beta(x_0,u_0)-1)}{\eta(x_0,u_0)^{\beta(x_0,u_0)}(x_0,u_0)} \end{cases} \tag{8}$$

These relations allows us, by using usual techniques of Weibull law parametrisation (see [7]), to define point by point the unknown operating condition functions $n(x,u)$ and $\psi(x,u)$, from various constant operating conditions, i.e. for some equilibrium points of the dynamical state space model. The model (6) thus obtained should make it possible to give in real-time a realistic estimate of the failure rate. Indeed, it allows us to take into account of the past and present environments under which a given component has evolved. The main difficulty of this approach is to find an estimate of the unknown operating functions $n(x,u)$ and $\psi(x,u)$. This problem is discussed in the following section.

3 Approximation of the operating functions

Consider l operating conditions (x_0^i,u_0^i), $i = 1,\cdots,l$. For convenience, we denote $\xi(t)$ the instantaneous operating condition i.e. $\xi(t) = [x(t) \quad u(t)]^T$, and ξ_i a particular constant operating condition, that is $\xi_i = [x_0^i \quad u_0^i]^T$. Let Ξ be the set of considered operating conditions :

$$\Xi = \{\xi_1,\xi_2,\cdots,\xi_l\} \tag{9}$$

For a considered component of the system, and for each operating condition, we assume that we have the life-time data.

$$\mathcal{L}_i = \{t_1^i,t_2^i,\cdots,t_{n_i}^i\} \tag{10}$$

these data can then be used for the determination of the parameters of the Weibull law for the operating condition characterized by ξ_i :

$$\mathcal{L}_i \rightarrow (\beta(\xi_i),\eta(\xi_i)), \quad i = 1,\cdots,l \tag{11}$$

The detail of the procedure on how to obtain $\beta(\xi_i)$, $\eta(\xi_i)$, from $\mathcal{L}_i$, is well known and is therefore omitted (on can see [7] for the details). Now, using relation (8) gives the corresponding values of the operating functions, i.e. $n(\xi_i)$ and $\psi(\xi_i)$. Finally, the functions $n(\xi(t))$ and $\psi(\xi(t))$ are known for a finite number of points. Starting from these values, it is possible to construct a staircase approximation of the continuous functions $n(\xi(t))$ and $\psi(\xi(t))$, as shown figure 1.

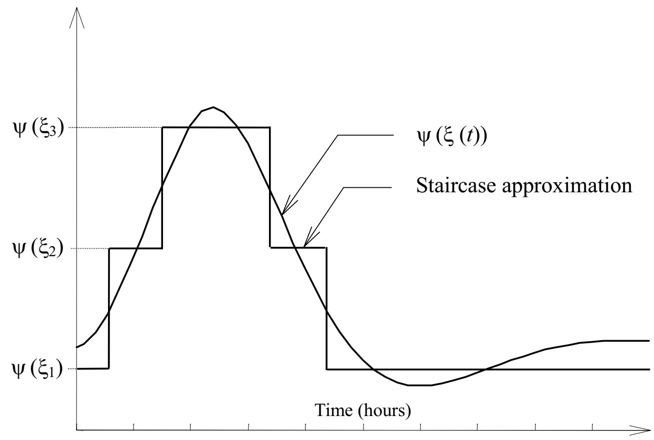

FIG. 1 – *Staircase approximation of the operating function* $\psi(\xi(t))$.

This approximation can be built by seeking the point $\xi_i \in \Xi$, nearest of $\xi(t)$. The nearest point $\xi_i \in \Xi$ of $\xi(t)$ can be found by solving:

$$z(t) = \arg\min_{\xi_i \in \Xi} ||\xi(t) - \xi_i|| \tag{12}$$

where $||\bullet||$ denotes the euclidian norm of $\bullet$. In these conditions we have $\psi(z(t)) = \psi(\xi_i)$ and $n(z(t)) = n(\xi_i)$, if $\xi(t)$ is about ξ_i, for $i = 1, \cdots, l$. What means that we obtain thus a function which is switched between known values accordingly to the observed operating condition. The complete dynamic failure-rate model is then given by :

$$\begin{cases} \dot{\lambda}(t) = t^{n(z(t))}\psi(z(t)) \\ \dot{x}(t) = f(x(t),u(t)) \\ z(t) = \arg\min_{\xi_i \in \Xi} ||\xi(t) - \xi_i||, \quad \xi(t) = [x(t) \quad u(t)]^T \\ \xi_i = [x_0^i \quad u_0^i]^T, \quad f(x_0^i, u_0^i) = 0 \end{cases} \tag{13}$$

Note that this model requires the measurement of all state variables. However, the availability of these variables for direct measurement is a rare occasion in practice. In most cases there is a true need for reliable estimation of the unmeasurable (or unmeasured) state variables, especially when they are used for process monitoring purpose as is our case here. For this particular task, a state observer is usually employed, in order to accurately reconstruct the state variables of the process with the available measurements. In the case of linear systems, the observer design theory developed by Lunberger [6], offers a complete and comprehensive answer to this problem. The classical Luenberger observer can be extended to nonlinear systems via extended linearization [1]. Following this approach, consider the nonlinear model of the process (3), and the following output equation reflecting the measured variables :

$$y(t) = Cx(t) \tag{14}$$

where $y \in \mathbf{R}^{n_y}$ is the vector of measured variables and $C \in \mathbf{R}^{n_y \times n_x}$ is known constant matrix. For this system we consider the nonlinear state estimator:

$$\begin{cases} \dot{\hat{x}}(t) = f(\hat{x}(t),u(t)) + g(y(t),u(t)) - g(\hat{y}(t),u(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{15}$$

where $\hat{x}$ is the state estimation, $\hat{y}$ is the estimated output and $g(.,.)$ is a nonlinear function which must be determined in order to ensure $\lim_{t\to+\infty} x(t) - \hat{x}(t) = 0$. The details for the determination of the function $g(.,.)$ are given in [1]. Using this state estimator, the dynamic failure rate model is now given by :

$$\begin{cases} \dot{\lambda}(t) = t^{n(\hat{z}(t))}\psi(\hat{z}(t)) \\ \dot{\hat{x}}(t) = f(\hat{x}(t),u(t)) + g(y(t),u(t)) - g(\hat{y}(t),u(t)) \\ \hat{y}(t) = C\hat{x}(t) \\ \hat{z}(t) = \arg\min_{\xi_i \in \Xi} ||\hat{\xi}(t) - \xi_i||, \quad \hat{\xi}(t) = [\hat{x}(t) \quad u(t)]^T \\ \xi_i = [x_0^i \quad u_0^i]^T, \quad f(x_0^i, u_0^i) = 0 \end{cases} \tag{16}$$

Finally, the failure rate at time t, taking into account the past time-varying operating condition is given by:

$$\lambda(t) = \int_0^t \dot{\lambda}(\sigma)d\sigma \tag{17}$$

This information can then be used to predict reliability under a given future operating condition. This aspect is discussed in the next section.

4 Reliability prediction in operation RPO

Reliability is the ability of a system or component to perform its required functions under stated conditions for a specified period of time. This ability is usually evaluated in terms of probability and denoted by $R(t)$. The reliability function and the failure rate are related as:

$$R(t) = \exp\left(-\int_0^t \lambda(\sigma)d\sigma\right) \tag{18}$$

Reliability prediction in operation (RPO) can be defined as the conditional probability that a system will be operating properly at a given future time $t+\tau$, where t is the current time, $\tau \in [0, \tau_h]$ and τ_h the horizon prediction. This conditional probability can be written as:

$$R(t+\tau|t) = \frac{R(t+\tau)}{R(t)} = \frac{\exp\left(-\int_0^{t+\tau} \lambda(\sigma)d\sigma\right)}{\exp\left(-\int_0^{t} \lambda(\sigma)d\sigma\right)} = \exp\left(-\int_t^{t+\tau} \lambda(\sigma)d\sigma\right) \tag{19}$$

Note that $\lambda(t+\tau)$ is unknown for $\tau > 0$, but it can be estimated assuming a given future operating condition, i.e.:

$$R_i(t+\tau|t) = \exp\left(-\int_t^{t+\tau} \lambda_i(\sigma)d\sigma\right) \tag{20}$$

where λ_i is the failure rate associated to the i^{th} operating condition ($i = 1, \cdots, l$).

5 Numerical example

We consider a device modeled by the following state space equation:

$$\dot{x}(t) = \frac{1}{T}(u(t) - x(t)) \tag{21}$$

where u is the input ($u \in [0, 5]$), T is the time constant ($T = 1$ hour) and x is the state variable. At the equilibrium, we have $x_0 = u_0$. The measurement of the state variable is then sufficient to characterize the operating condition.

OC	LTD	$\beta(x_0)$	$\eta(x_0)$	$n(x_0)$	$\psi(x_0)$
$x_0 \in [0, 1.5]$	$\mathcal{L}_1 = \{238,246,231,103,166\}$	4.62	217	2.62	2.70×10^{-10}
$x_0 \in (1.5, 3]$	$\mathcal{L}_2 = \{222,359,87,154,66\}$	1.79	201	-0.25	1.07×10^{-4}
$x_0 \in (3, 5]$	$\mathcal{L}_3 = \{175,139,92,210,73\}$	3.00	155	1.00	1.60×10^{-6}

Tab. 1 - *Parametrization of the reliability model.*

In this example, we consider 3 operating conditions for which life time data are available. Table 1 summarize the parametrization procedure presented in section 3. For the simulation, we apply to the input of this device successive step changes so that the system evolves into its entire operating range. Figure 2 shows the corresponding measured output $x(t)$. This measurement, is applied to the dynamic model (13) which allows us to estimate on-line, the failure rate and the reliability (see figure 2).

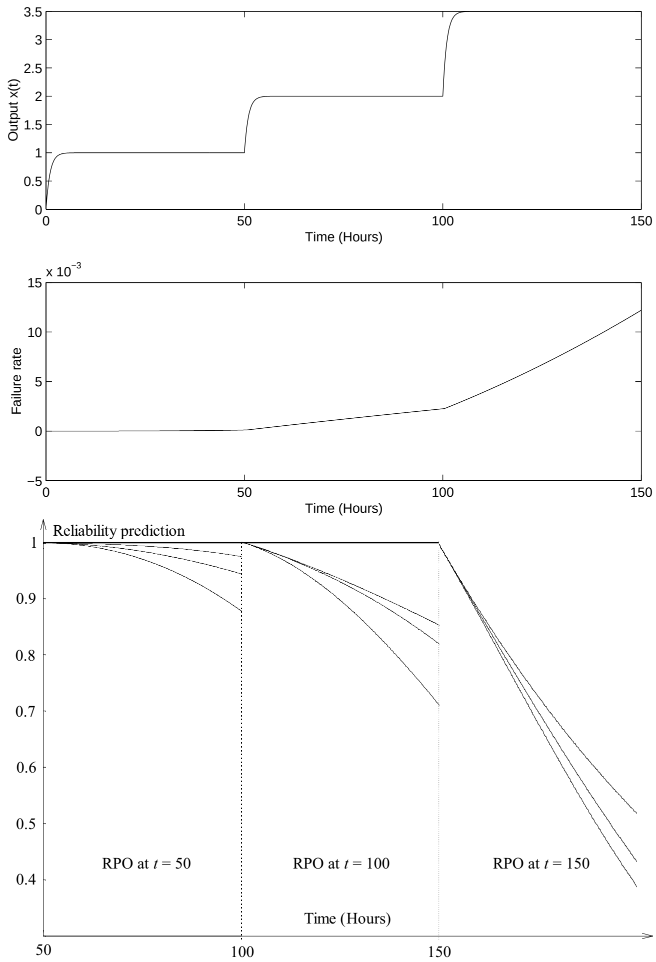

FIG. 2 – *Output response of the device, failure rate & reliability prediction.*

Figure 3 shows the details of the reliability prediction in operation for the instants : $t = 50$, $t = 100$ and $t = 150$.

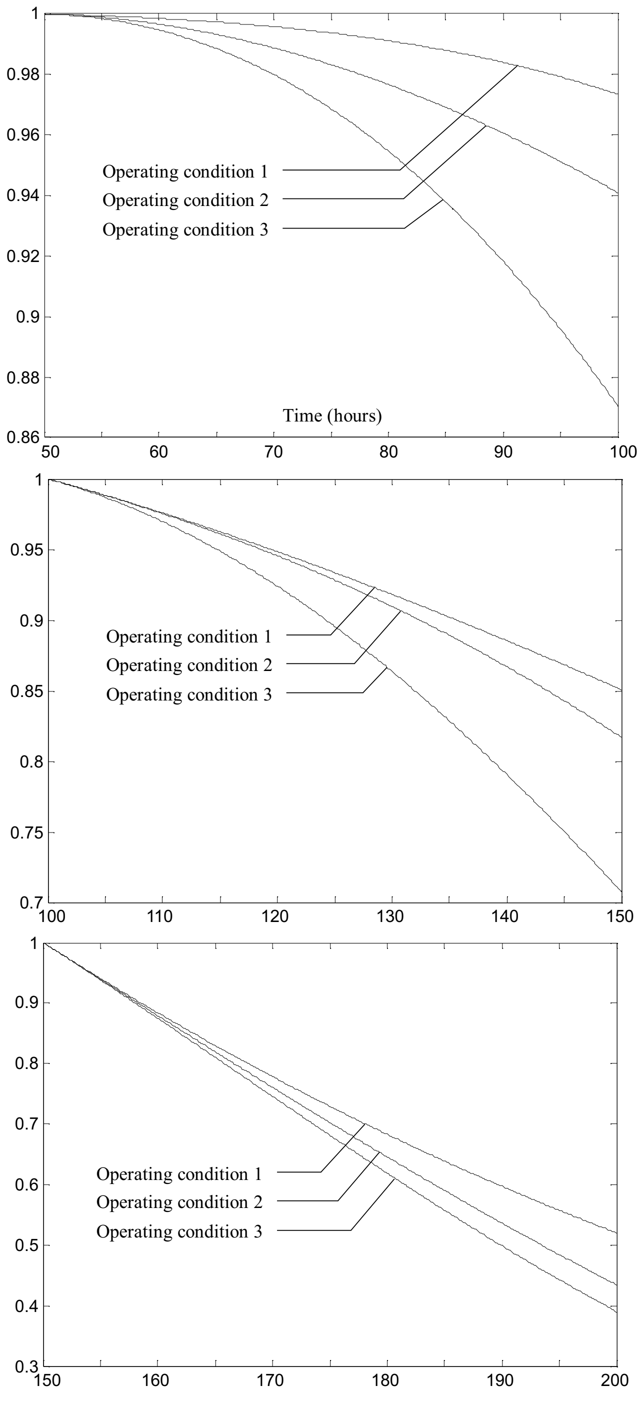

FIG. 3 – *Details of the reliability prediction at time* $t = 50$, $t = 100$, *and* $t = 150$.

From these results, one can see, for instance, that the probability that the device works properly, at time 100 hours, after it survived at time 50 hours, is 0.885 under operating condition 3. Given the device survived at time 100 hours, the probability that it works at time 150 hours is 0.71 under operating condition 3. Similarly, the probability that the device works at time 200 hours, after it survived at time 150 hours, is 0.39 under operating condition 3. Similar observations can be done for other operating conditions.

6 Conclusion

The classical reliability method enables us to model the reliability of components using life-time data provided, for instance, by a return of experiments. However, the reliability model thus obtained is valid only under the operating condition around which the component has evolved. The reliability of a component will change under different operating conditions. In order to make reliability prediction over a wide range of operating conditions, we have proposed a dynamic reliability model able to take into account the history of process running. This was done by using a differential model of failure rate combined with the state space representation of the process. Simulation studies have shown the practical applicability of this new concept.

Références

[1] W. T. Baumann. Feedback control of multiinput nonlinear systems by extended linearization. *IEEE Trans. Automatic Control*, 33:193–197, 1988.

[2] M.B. Carey and R.H. Koening. Reliability assessment based on accelerated degradation: a case study. *IEEE Trans. Reliability*, 40:499–560, 1991.

[3] C.J. Lu and W.Q. Meeker. Using degradation measures to estimate a time-to-failure. *Technometrics*, 35:161–174, 1993.

[4] H. Lu, W.J. Kolarik, and S.S. Lu. Real-time performance reliability prediction. *IEEE Trans. Reliability*, 50:353–357, 2001.

[5] S. Lu, H. Lu, and W.J. Kolarik. Multivariate performance reliability prediction in real-time. *Reliability Engineering & System Safety*, 72:39–45, 2001.

[6] D. G. Luenberger. Observing the state of a linear system. *IEEE Transaction on Military Electronics*, 8:74–80, 1964.

[7] P. Lyonnet. *Maintenance Planning*. Chapman & Hall, London, 1991.

[8] W. Nelson. Analysis of performance-degradation data from accelerated tests. *IEEE Trans. Reliability*, 30:149–155, 1981.

[9] J. Xue and K. Yang. Upper and lower bounds of stress-strength interference reliability with random strength-degradation. *IEEE Trans. Reliability*, 46:142–145, 1997.