

A new stochastic inverse identification of the mechanical properties of human skin

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Abstract

The study of the human skin mechanical properties is a key point to understand better surgery, skin ageing and pathologies. Skin is a complex medium which is made up of three layers: the epidermis, the dermis and the hypodermis. This composite-like structure presents an anisotropic nonlinear quasi incompressible viscoelastic behaviour which is essentially due to the peculiar composition of the dermis. Moreover, as the skin is a living tissue, it must be studied in vivo, hence analytical solutions are really difficult to obtain. In this study we propose a new stochastic inverse method for the identification of the mechanical properties of the skin.

The optimization method we developed is first presented. It is based on an iterative stochastic approach which ensures the identification of a global extremum. An actual case study is then analysed: the suction experiment. As analytical solutions of this test are not well-known, an inverse method which is based on comparisons between experimental data and finite element models is presented. The elastic components of the skin are only considered. The solutions for the Recursive Least Squares and Gauss-Newton's problems are finally compared with the proposed method to conclude on this study and to briefly present our future works.

1 Introduction

The mechanical properties of living tissues are of potential interest in the identification of certain diseases, for assessing therapeutic intervention, or for predicting the effect of trauma. Due to its unusual structure, the skin presents a nonlinear viscoelastic anisotropic quasi incompressible mechanical behaviour which has to be studied *in vivo* (Agache *et al.* [1]). To identify the skin mechanical properties, complex numerical models which lead to multi-parameter optimization problems are usually studied. The optimization algorithms thus need to reach the solution despite the local minima and stabilization problems. Moreover, as clinical studies are performed with a large number of measurements, the mechanical parameters need to be identified rapidly.

The developed method aimed at identifying the mechanical properties of the skin through comparisons between experimental results and finite element models. A suction deformation test is performed on the volar aspect of the forearm of a subject. A finite element model of this test is then created. The optimization process is finally used so as to minimize the difference between the experimental and the numerical results. To underline the

usefulness of the proposed method, comparisons with standard Gauss-Newton's and Recursive Least Squares methods are finally drawn for an actual identification case.

2 Methods

The proposed method is divided into three stages. First the suction experiment is performed and modeled through the finite element method. A numerical data base which is made up of simulated curves that are obtained for different values of the mechanical parameters is then created.

2.1 The suction experiment

The *in vivo* suction test (Diridollou *et al* [2]) consists in applying a negative pressure at the skin surface. In our case study, this test is performed on the volar aspect of forearm skin, using a Cutometer CM570 (Courage & Khazaka, Cologne, Germany). The skin is sucked into an aperture forming a dome whose deflection is recorded for each step of pressure p . Figure 1.a presents an actual curve which was obtained for a pressure up to 100mbar, applied at a rate of 20mbar.s⁻¹.

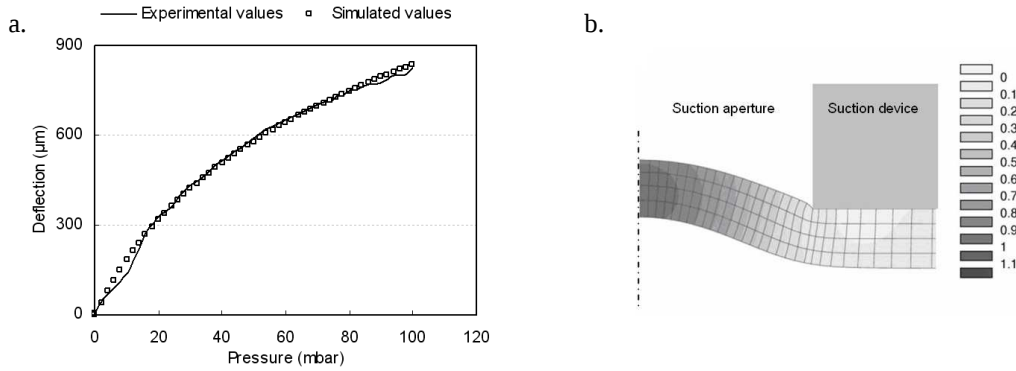


Figure 1: The suction test. a. The experimental curve. b. The finite element numerical model (The vertical displacements (mm) are presented).

2.2 The finite element model

The numerical simulations are performed with the SYSTUSTM finite element software. To model a complex structure such as skin, several assumptions need to be made. The skin is considered to be a homogeneous medium, its structural response is only considered here. Due to the low stress rate, the mass inertial effects are neglected and computations are thus performed using a quasi-static calculation. The geometry is considered to be axisymmetrical (Figure 1.b) and the skin was modeled as a single layer which thickness is the one corresponding to the epidermis and the dermis. Indeed, results from literature clearly state that the effects of sub-dermal structures can be neglected (Diridollou *et al.* [2]), (Cook *et al.* [3]). In contrast to studies that account for the viscoelastic and the anisotropic behaviour of the skin (Khatyr *et al.* [4]), our approach consists in identifying its isotropic elastic mechanical properties. Second order elements with a reduced integration scheme so as to avoid volumetric locking are used for the mesh.

A compressible hyperelastic law, based on an extended neo-Hookean potential (Hendricks *et al.* [5]) was developed (Equation 1). X_1 and X_2 are the elastic parameters, J_i denotes the i^{th} reduced invariant of the right Cauchy-Green deformation tensor $\mathbf{C}$, and κ the compressibility constant of the media. For small strains, a relationship can be drawn between the compressibility constant and Poisson's ratio ν : $\kappa = 6(1-\nu)X_1 / ((1+\nu)(1-2\nu)) - 8/3 X_1$. Three mechanical parameters thus need to be identified: X_1 , X_2 and ν .

$$W = X_1 (J_1 - 3) + X_2 (J_1 - 3)(J_2 - 3) + \frac{\kappa}{2} (J_3 - 1)^2 \quad (1)$$

2.3 The inverse method principle

The main principle of the proposed method lies in the construction of a pre-calculation data base: the *simulated space*. The results of several numerical calculations, relating to the different combinations of the mechanical parameters to be determined, are collected in this data base. Indeed, clinical studies are performed with a large number of measurements. With the use of standard inverse methods, a finite element computation is required for each identification step. Redundant solutions to the numerical models are thus calculated. In the present procedure, all the finite element models are performed only one time. Furthermore such experimentation is usually performed by medical practitioners, hence our method needs to be a FEM-free process. The variation field of the simulated space is: $X_1(\text{Mpa}) \in [0.01; 0.1]$ per step of 0.01MPa, $X_2(\text{Mpa}) \in [0.1; 4.9]$ per step of 0.3MPa, $\nu \in [0.2; 0.48]$ per step of 0.04 and $p(\text{mbar}) \in [0; 100]$ per step of 2mbar.

The iterative optimization algorithm is then used to minimize the square identification error related to the comparison of simulated and experimental curves. During the calculation, non-simulated values can be identified. They are simply calculated with cubic Lagrange's interpolations of the existing data.

2.4 The stochastic algorithm fundamentals

Many engineering problems can be solved through optimization procedures, i.e. can be seen as the act of achieving the best possible result under given circumstances. The goal of optimization is either to minimize effort or to maximize benefit which can usually be expressed as a function of specific design variables. Hence optimization is the procedure which searches for the conditions that give the maximum or the minimum value of a particular function called objective function or cost function.

In this article a heuristic method to solve the following optimization problem is proposed:

$$X_{opt} = \arg \min_{x \in X} J(x) \quad (2)$$

Without requiring the derivatives of the cost function J and without necessarily requiring its mathematical expression (i.e. the values of J can be obtained through simulations or experiments). The set $X \subset \mathbf{R}^n$ represents the admissible solutions.

To solve this problem, we adopt the principle depicted in figure 2. The proposed procedure is iterative, and we denote by j , the j^{th} iteration of the algorithm.

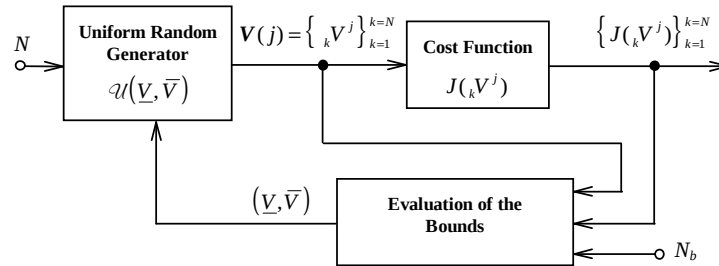


Figure 2: Principle of the algorithm.

A uniform random generator is used, which produces, at each iteration, a sequence of N vectors that are uniformly distributed between the bounds $\underline{V} = [\underline{v}_1 \ \dots \ \underline{v}_n]^T$ and $\bar{V} = [\bar{v}_1 \ \dots \ \bar{v}_n]^T$:

$$V(j) = \{ {}_1V^j, {}_2V^j, \dots, {}_NV^j \} \quad (3)$$

where ${}_kV^j$ is the k^{th} vector generated at the iteration number j : ${}_kV^j = [{}_kV_1^j \ \dots \ {}_kV_n^j]^T$, and ${}_kV_l^j$ is the l^{th} component of ${}_kV^j$ ($l = 1, \dots, n$). This uniform random generator is applied to the cost function J . Without loss of generality, we assume that the vectors are ordered by their increasing cost function i.e. :

$$J({}_1V^j) < J({}_2V^j) < \dots < J({}_NV^j) \quad (4)$$

The principle of the algorithm is to modify the bounds $\underline{V}$ and $\bar{V}$ of the uniform random generator until the minimum of the cost function is reached. Obviously, the minimum is characterized by a difference between the bounds close to zero. The given problem is: how to modify the parameters of the uniform random generator in order to achieve the optimum?

To solve this problem we consider N_b best candidates i.e. N_b candidates that are the more representative of the optimum. For the iteration j , the bounds $\underline{V}$ and $\bar{V}$, of the uniform random generator are then evaluated as follows :

$$\begin{cases} \underline{v}_l = \min_{1 \leq k \leq N_b} v_l^j \\ \bar{v}_l = \max_{1 \leq k \leq N_b} v_l^j \end{cases}, \quad l = 1, \dots, n \quad (5)$$

where N_b is the number of considered candidates. This procedure is repeated until the euclidian norm of the difference $\bar{V} - \underline{V}$ is small enough. The result obtained with this stochastic algorithm is not an exact optimum but a near optimum which is closely related to number of samples N . The minimum number of samples required to obtain a reliable near optimum, must be chosen as (Toscano [6]) (Toscano *et al.* [7]):

$$N \geq \ln(1 - \rho) / \ln(1 - e) \quad (6)$$

where ρ is the confidence (ρ must be close to 1 e.g. $\rho = 0.99$) and e is an accuracy (e must be close to 0 e.g. $e = 0.01$).

2.5 Implementation of the stochastic approach

The fundamentals of the stochastic algorithm were implemented in three stages. First, the **Maximal** and **minimal** boundary parameters $^M X_1^0$, $^m X_1^0$, $^M X_2^0$, $^m X_2^0$, $^M v^0$ and $^m v^0$ are defined. They usually correspond to the boundary values of the simulated space (Figure 3 step 0). N values of this domain are then randomly chosen and, for each iteration j , their related solutions $S(X_1^j, X_2^j, v^j)$ with $k = 1, \dots, N$ are calculated thanks to the simulated space $S(X_1^j, X_2^j, v^j) = S(X_1^j, X_2^j, v^j, p_i)$ (Figure 3 step 1). They are finally sorted according to the quadratic error E_j which is defined in Equation 7, where u is the number of experimental points and $M = M(p_i)$ are the experimental measurements. The minimal and maximal values of the N_b firsts sorted results are finally selected for each mechanical parameters, to be the starting point of another iteration (Figure 3 step 2).

$$E_j = \frac{1}{u} \left(\sum_{i=1}^{i=u} (M(p_i) - S(X_1, X_2, v, p_i))^2 \right)^{1/2} \quad (7)$$

3 Results

To underline the usefulness of the proposed method, comparisons are drawn with the results of the standard Gauss-Newton (*GN*) (Björck, [8]) and Recursive Least Squares (*RLS*) methods (Omid, [9]). To improve its reliability, the *RLS* technique was developed thanks to a forgetting parameter approach ($\lambda=0.9$) (Omid, [9]) and was modified through a stabilization process which consists in running the calculations several folds on the same curve (Delalleau, [10]). For both the *GN* and *RLS* methods, the initial parameters are equal to the smallest boundary values of the simulated space. The calculations were performed on a Pentium core-duo 1.6GHz.

To check the relevance of the stochastic algorithm (*STO*) we performed a test which consists in using curves extracted from the simulated space (with known mechanical parameters) as if they were experimental ones (Delalleau, [10]). For each case study, the obtained results are in accordance with the required parameters and show an error value $E < 0.1\mu m$.

Table 1 presents the results corresponding to the experimental curve (Figure 1.a). For that case study, the stochastic approach shows the best results whatever the value of N is. One can note that even if N presents high values, the computation time can decrease if less iterations are needed to reach the convergence. Figure 1.a presents a comparison between the experimental and the simulated deflection obtained for the identified parameters. It shows that the results are in accordance. The *GN* algorithm stabilizes to erratic parameters and

presents a high computation error. The *RLS* analyse is more unusual. The calculation cannot stabilize for the specified convergence criterion, however, for higher values (i.e. $C = 10^{-2} \mu\text{m}$ with $C_j = |E_{j-1} - E_j|$) the convergence can be reached with a low computational error ($E=2.1\mu\text{m}$). Indeed the *RLS* convergence shows oscillating values around the solution. A line-search technique should thus be used for the *GN* and *RLS* computations whereas the stochastic approach does not need any.

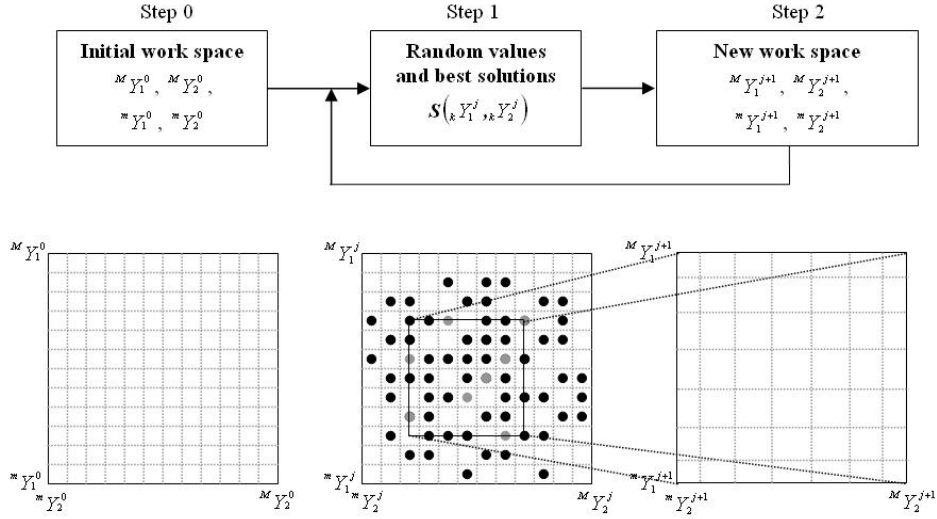


Figure 3: Implementation of the stochastic approach for $N = 67$, $N_b = 8$ and two parameters to be identified : Y_1 and Y_2 .

Several sets of initial values were tested to study the sensitivity of both the *GN* and the *RLS* algorithms to the computation initialization. In every case these algorithms do not reach satisfactory solutions. The relevance of the stochastic method was also tested for 50 computations which correspond to different random samples (Figure 4). The results show that for an identical error value, the identified parameters always present the same order of magnitude. For $N=10^4$ closest variations could be observed. They would finally tend to 0 if N tends toward the infinity, as the global minimum is reached. Finally, the influence of the N_b chosen parameters was also studied. For values under 10, the algorithm can reach non reliable parameters whereas for above values, no effect of this parameter was noticed.

Method	X_1^{id} (MPa)	X_2^{id} (MPa)	v^{id}	CPUT(s)	$E(\mu\text{m})$
<i>GN</i>	0.016	0.1	0.48	3.2	43
<i>RLS</i>	/	/	/	/	/
<i>STO</i> $N=10^2$	0.025	2.95	0.28	2.3	2
<i>STO</i> $N=10^3$	0.026	3.1	0.26	1.4	2
<i>STO</i> $N=10^4$	0.025	3.06	0.26	15.2	2

Table 1: Identified values for an actual case study, $C = 10^{-5} \mu\text{m}$, and $N_b = 20$.

4 Discussion

In this article, a new stochastic approach which ensures the identification of a global minimum was developed and successfully compared with the Gauss-Newton and Recursive Least Squares algorithms. In our case study, the *GN* identification usually reaches the boundary values of the simulated space and the *RLS* approach oscillates around the solution. This was also tested for several other calculations. In most of the cases, identical

conclusions can be drawn. Both these processes are known to be sensitive to the initial values and, without a particular line-search technique, they may oscillate around local extrema or diverge. For each iteration, a specific optimization procedure is thus required. Moreover, these standard methods necessitate to know the cost function of the problem and to evaluate its derivatives. Time and memory consuming calculations which are not required by the stochastic approach are thus performed.

The main advantages of the developed method lie in its rapidity to converge and in its relevance. Indeed, many parameters can be identified easily without neither computational divergence nor oscillating values. Further more, no initial set of values, which constitutes a critical stage for most of the optimization processes, is required. The stochastic method is theoretically able to reach the global minimum of the problem. However, this requires an infinite number N of random samples, hence, the obtained results should be interpreted in terms of probability. These properties are currently used to identify the mechanical parameters of the human skin through more complex behaviour laws and multi-layered models. These works will be presented in a future paper.

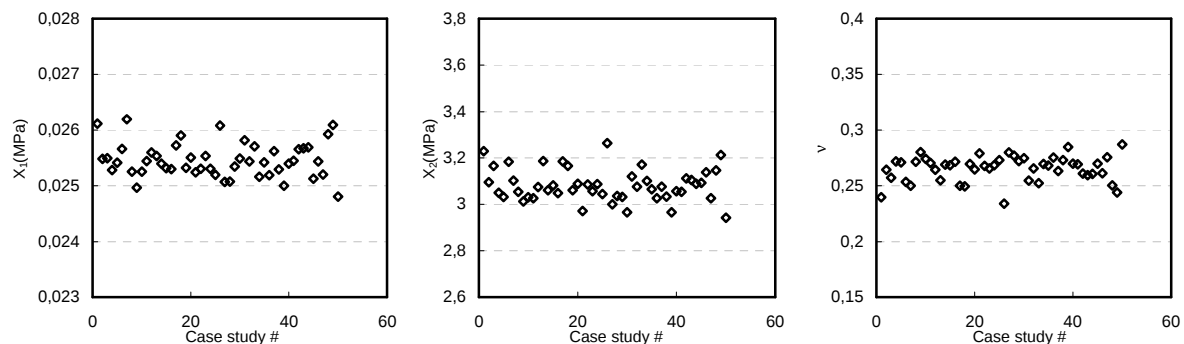


Figure 4: Identified mechanical parameters for the stochastic technique according to 50 random cases.

$$C = 10^{-5} \mu\text{m}, N_b = 20, N = 10^3.$$

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